Orthogonal Perfect DFT Golay Codes

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Abstract Deciding about which coding sequences should be used is an important issue in many communication systems, such as those employing Code Division Multiple Access (CDMA). In this paper, we propose a cascading electronic codec based on a recursive algorithm for the generation of perfect sequences, derived from an Inverse Discrete Fourier Transform (IDFT) of Golay codes. The sequences generated have a low Peak-to-Average Power Ratio (PAPR), around 3 dB, which is the PAPR of a sine wave. The proposal is compared to the original Golay codes, verifying that our codec provides better crosscorrelation and autocorrelation results, crucial to gain immunity to multi-path interference.

1 Introduction

Many communication systems, such as those employing Code Division Multiple Access (CDMA), require appropriate coding sequences, which should have a perfect periodic autocorrelation and excellent crosscorrelation properties for synchronization or code detection in noisy environments. Some well-known orthogonal codes used for CDMA are Golay codes [1], Frank and Chu perfect sequences [2], and Gold codes [3] [4].

It is well-known that perfect sequences are complex sequences, having all out-of-phase periodic autocorrelation values equal to zero. Unfortunately, perfect bipolar sequences of length greater than 4 and perfect quadri-phase sequences of length greater than 16 are unknown [5].

In this paper, we propose an encoder of perfect sequences of length 2^N, with $N \in \mathbb{N}$, which are derived from an Inverse Discrete Fourier Transform (IDFT) of Golay codes. The codes obtained are complementary perfect sequences [6] with a low Peak-to-Average Power Ratio (PAPR). Because of their correlation properties they are immune to Multi-Path Interferences (MPI) [6]. These sequences are called Orthogonal Perfect DFT Golay (OPDG) codes [7].

2 OPDG generator

Figure 1 illustrates a simplified OPDG encoder, based on a discrete constant input signal A, generating two OPDG codes a_N and b_N. Those codes are then converted to analog signals by the Digital-to-Analog Converter (DAC) and sent through the physical transmission medium, which will contaminate them with noise. The new codes are received and converted to digital by the Analog-to-Digital Converter (ADC). Finally, the OPDG decoder recovers the original input signal A.

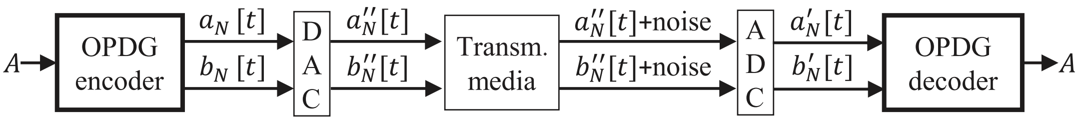

Figure 1: Generic OPDG application diagram.

The OPDG encoder (Figure 2, see next page) generates a pair of perfect orthogonal sequences, $a_N[t]$ and $b_N[t]$, of size $L = 2^N$. According to Figure 2, the encoder is composed of N cascading basic modules, each one with an adder, a subtractor, and a multiplier. Thus, given two complex input vectors $a_{n-1}[t]$ and $b_{n-1}[t]$ $n \in 1, \ldots, N$, the basic module generates two new values $a_n[t]$ and $b_n[t]$ given by:

$$a_n[t] = a_{n-1}[t] + q \cdot W_L^{-t \cdot 2^{n-1}} \cdot b_{n-1}[t] \tag{1}$$

and

$$b_n[t] = a_{n-1}[t] - q \cdot W_L^{-t \cdot 2^{n-1}} \cdot b_{n-1}[t], \tag{2}$$

where $q = \pm 1$, t is the temporal displacement of the sequences, and with initial conditions $a_0[t] = A$ and $b_0[t] = A$, where A is a constant sequence of L discrete values, all of them assuming values 1 or -1. Let W_L be the twiddle factor

$$W_L = \exp\left(-j \cdot \frac{2\pi}{L}\right), \tag{3}$$

where j is $\sqrt{-1}$.

An important parameter of a signal in wireless communication systems is the Peak-to-Average Power Ratio. As it is well-known, this ratio affects the power amplifiers efficiency and cost. The PAPR is defined as the peak signal amplitude squared divided by the root mean square of the signal squared. It can be calculated (in dB) with

$$\text{PAPR} = 10 \cdot \log_{10}\left(\frac{|x|_{peak}{}^2}{x_{rms}{}^2}\right), \tag{4}$$

where x is the complex modulus of the OPDG code. Applying (4) to a_N and b_N gives results very similar to the sine wave PAPR: ≈ 3.01 dB. We have calculated the PAPR for OPDG sequences for $N \in 3, \ldots, 20$ and the result remains stable around 3 dB. The only exception is with $N = 4$, where the PAPR is even lower (2.3 dB).

The OPDG decoder (Figure 3) implements an autocorrelation function. If the input sequences are correct, i.e. $a'_N[t] = a_N[t]$ and $b'_N[t] = b_N[t]$, the decoder generates two outputs: an autocorrelation function proportional to the Dirac impulse $\delta(t)$, and a constant function proportional to the input signal A of the encoder. Otherwise, a null crosscorrelation function is obtained. The OPDG decoder is also composed of N basic modules. In each module, given two input vectors, $a'_n[t]$ and $b'_n[t]$, the module calculates two new values $a'_{n-1}[t]$ and $b'_{n-1}[t]$, as follows

$$a'_{n-1}[t] = q \cdot W_L^{-t \cdot 2^{n-1}} \cdot (a'_n[t] + b'_n[t]) \tag{5}$$

and

$$b'_{n-1}[t] = a'_n[t] - b'_n[t]. \tag{6}$$

The last block of Figure 3 implements the real part of a Fast Fourier Transform (FFT), providing a Dirac impulse of amplitude $A[q]^N 2^{2N+1}$ with a shift of $2^N - 1$.

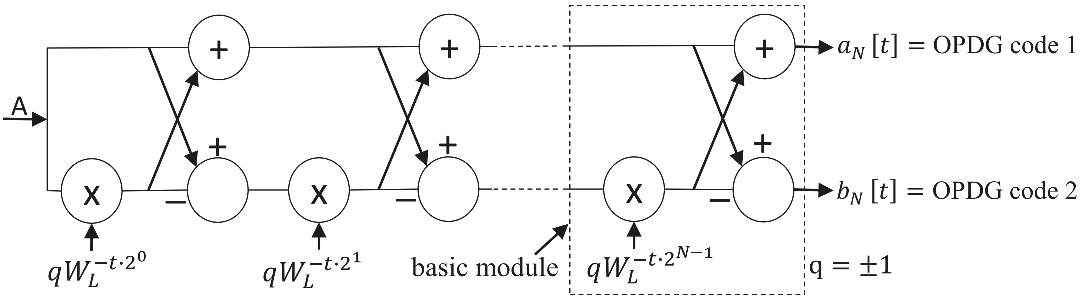

Figure 2: OPDG encoder.

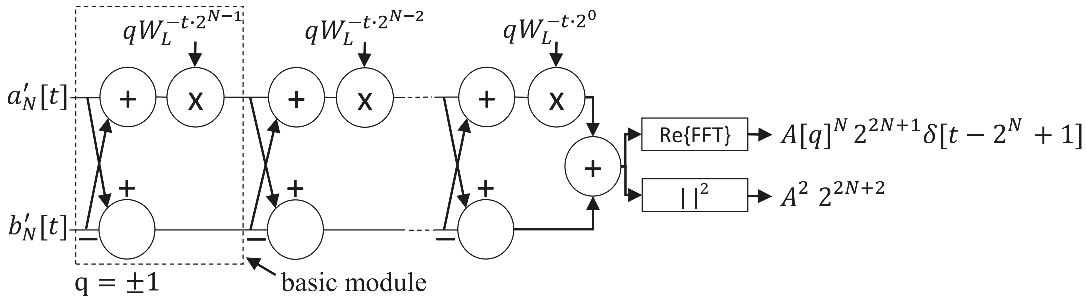

Figure 3: OPDG decoder.

3 Comparisons with other approaches

In this section, we compare our proposal to the well-known Golay codes. We generated a pair of bipolar sequences of length 2^N, where N equals 5, with the aim of analysing the behavior of the codes. We consider both autocorrelation and crosscorrelation functions.

Figure 4 (see following page) shows periodic autocorrelation functions for OPDG 1 and 2 [7], Golay 1 and 2 [1]. As shown, the OPDG codes provide a perfect periodic autocorrelation, while Golay codes have out-of-phase peaks. Since each OPDG code is complex valued, it can be split into real part $\text{Re}(OPDG1)$ and imaginary part $\mathrm{Im}(OPDG2)$. These two sequences are complementary orthogonal sequences. The crosscorrelation properties of the generated codes are illustrated in Figure 5. As this plot shows, the crosscorrelation of $\mathrm{Re}(OPDG1)$ with $\mathrm{Im}(OPDG1)$, as well as the crosscorrelation of $\mathrm{Re}(OPDG2)$ with $\mathrm{Im}(OPDG2)$, is always zero, regardless of the phase and for any length (N). This does not happen with the crosscorrelation of Golay codes.

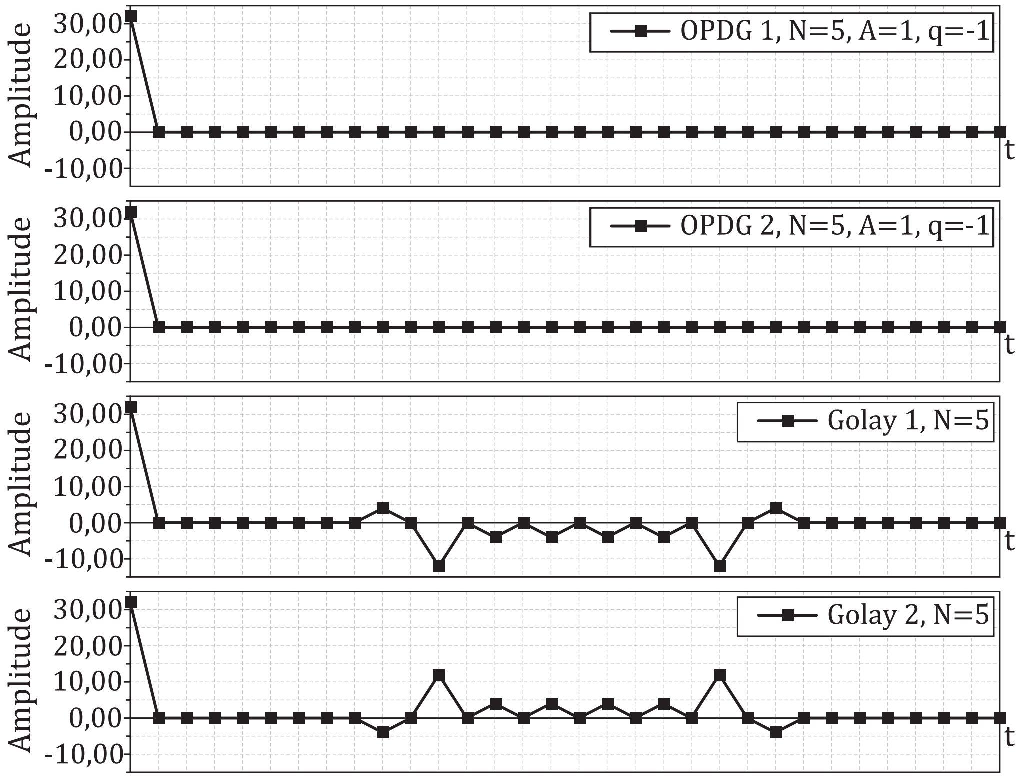

Figure 4: Periodic autocorrelation amplitude vs. code index t, for length 2^5.

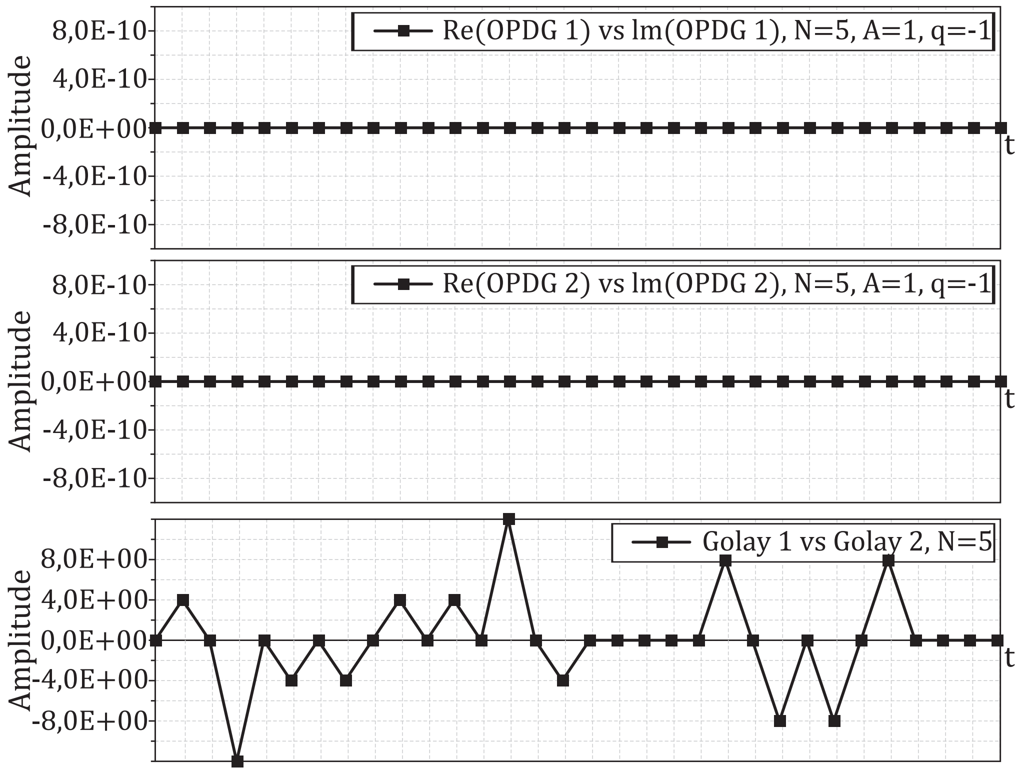

Figure 5: Periodic crosscorrelation amplitude vs. code index t, for length 2^5.

4 Conclusions

This letter proposes a novel codec that generates perfect sequences, which are based on Golay codes, through an IDFT. These sequences are compared with the original Golay codes, by checking through both crosscorrelation and autocorrelation functions. These new perfect sequences, with an autocorrelation proportional to a Dirac pulse, are theoretically immune to multi-path interference. Their real and imaginary parts are also orthogonal, with a constant zero crosscorrelation value between them. These properties indicate a better behavior of the OPDG codes, when compared with the original Golay codes.

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