

# A Stochastic Dynamic Programming Approach to Optimize Short-Rotation Coppice Systems Management Scheduling: An Application to Eucalypt Plantations under Wildfire Risk in Portugal

Liliana Ferreira, Miguel F. Constantino, José G. Borges, and Jordi Garcia-Gonzalo

**Abstract:** This article presents and discusses research with the aim of developing a stand-level management scheduling model for short-rotation coppice systems that may take into account the risk of wildfire. The use of the coppice regeneration method requires the definition of both the optimal harvest age in each cycle and the optimal number of coppice cycles within a full rotation. The scheduling of other forest operations such as stool thinning and fuel treatments (e.g., shrub removals) must be further addressed. In this article, a stochastic dynamic programming approach is developed to determine the policy (e.g., fuel treatment, stool thinning, coppice cycles, and rotation length) that maximizes expected net revenues. Stochastic dynamic programming stages are defined by the number of harvests, and state variables correspond to the number of years since the stand was planted. Wildfire occurrence and damage probabilities are introduced in the model to analyze the impact of the wildfire risk on the optimal stand management schedule policy. For that purpose, alternative wildfire occurrence and postfire mortality scenarios were considered at each stage. A typical *Eucalyptus globulus* Labill. stand in Central Portugal was used as a test case. Results suggest that the proposed approach may help integrate wildfire risk in short-rotation coppice systems management scheduling. They confirm that the maximum expected discounted revenue decreases with and is very sensitive to the discount rate and further suggest that the number of cycles within a full rotation is not sensitive to wildfire risk. Nevertheless, the expected rotation length decreases when wildfire risk is considered. FOR. SCI. 58(4):353–365.

**Keywords:** stochastic dynamic programming, wildfire risk, forest management, eucalypt, coppice system

WILDFIRE IS ONE OF THE MAIN THREATS for forests in the Mediterranean and in Portugal (Alexandrian et al., 2000, Pereira et al., 2006). Large-scale forest fires throughout Mediterranean countries (e.g., Portugal, Spain, Italy, and Greece) have substantially increased during the last few decades (Velez 2006). The need to address wildfire risk in forest management planning is evident and yet fire and forest management are currently performed mostly independently of each other in these countries (Borges and Uva, 2006).

During the past few decades substantial effort has been devoted to develop operations research techniques to optimize even-aged stand management (e.g., Brodie and Kao 1979, Kao and Brodie 1979, Kao 1982, Hoganson and Rose 1984, Buongiorno and Gilless 1987, Roise 1986, Pukkala and Miina 1997). Initially, the aim of most efforts was to schedule harvests to optimize stocking control without paying attention to risk. Several authors have proposed dynamic programming (DP) models to optimize thinning regimes and rotation lengths (e.g., Amidon and Akin 1968, Brodie et al. 1978, Brodie and Kao 1979, Kao and Brodie 1979, Kao 1982, Buongiorno and Gilless 1987, Arthaud and

Pelkki 1996). Hoganson and Rose (1984) also used DP to find optimal stand-level prescriptions within a forestwide lagrangian relaxation approach. DP is very useful for stand-level optimization because it helps avoid the problem of needing to enumerate and evaluate all possible management options (Hoganson et al. 2008).

The optimization of coppicing stands addresses other management options. It involves the simultaneous optimization of the age for each coppice cycle and of the number of harvests before a stand is reestablished (Díaz-Balteiro and Rodriguez 2006). Medema and Lyon (1985) used an iterative process to search for both the optimal coppice harvest age in each cycle and the number of cycles. Tait (1986) introduced DP to the coppice system optimization framework.

Díaz-Balteiro and Rodriguez (2006) extended the use of DP to optimize carbon sequestration in coppice systems and demonstrated the impact of timber, of carbon prices, and of discount rate on the duration and on the number of cycles. Nevertheless, these models did not examine the effect of risk on stand management scheduling (e.g., wildfire risk).

Martell (1980), Routledge (1980), and Reed and Errico (1985) pioneered the integration of wildfire risk in stand

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management scheduling. Martell (1980) and Routledge (1980) used a discrete-time framework and showed that, generally, risk reduces the optimal rotation age. Reed (1984) examined this impact within a continuous Faustmann framework. Other authors further developed the continuous stochastic rotation model to address the cases when stands may produce amenities (e.g., Reed 1993, Englin et al. 2000, Amacher et al. 2009). More recently, the internalization of the role of forest managers in mitigating catastrophic risk has been addressed by simultaneous optimization of rotation age and of risk mitigation decisions (e.g., Reed 1987, Thorsen and Helles 1998, Amacher et al. 2005, González et al. 2005). The literature reports the use of stochastic simulation (e.g., Dieter 2001) and nonlinear programming (e.g., Möykkynen et al. 2000, González et al. 2005) for that purpose. However, addressing the impact of risk on optimal coppice management scheduling has not attracted much attention. DP makes it possible to recognize the stochastic nature of the stand management scheduling problem (Norström 1975, Haight and Smith 1991, Gunn 2005). Díaz-Balteiro and Rodríguez (2008) used a Monte Carlo approach to simulate timber prices and discount rates within a DP framework.

Nevertheless, DP approaches to optimize stand-level management planning have been used predominantly within an anticipatory framework. Anticipatory models are used for deriving in advance optimal decisions over the whole rotation (Zhou et al. 2008). In this case, the DP solution defines the optimal path over a rotation. However, addressing risk and uncertainty suggests an adaptive framework in which decisions are made according to the state of the system. DP fits well into an adaptive framework because the backward recursion process provides information about the best management option at any state (Walters and Hilborn 1978, Hoganson et al. 2008). This means that if a change occurs as a consequence of a random event such as a wildfire, the forest manager just has to look up the DP solution to get the management policy that is best adapted to the new situation. Ferreira et al. (2011) took advantage of these DP features to determine adaptive optimal management policies for high forest stand-level planning. However, no such models are available to optimize stand-level management planning for coppice forests.

In this article, we propose a stochastic DP solution approach to optimize the short-rotation coppice systems management scheduling problem. It encompasses a deterministic stand-level growth-and-yield model and wildfire occurrence and damage models. An innovative stochastic DP network that may take into consideration coppice schedules (i.e., number of coppice cycles and cycle lengths) and fuel treatments to address the risk of wildfires is designed.

Eucalypt coppice stands (e.g., *Eucalyptus globulus* Labill.) are of great importance in Portugal. Eucalypt plantations extend over  $647 \times 10^3$  ha (National Forest Inventory 2005), corresponding to about 20.6% of the total forest area in Portugal. In this country, wildfires are the most severe threat to eucalypt plantations, which provide key raw material for the pulp and paper industry. Moreover, the impacts of wildfires are prone to increase as a consequence of climate change. Nevertheless, no models combining forest and fire management planning activities have been adopted

to optimize eucalypt stand-level decisions (Díaz-Balteiro et al. 2009). Thus, no models that might help forest managers address the risk of wildfire in management scheduling of eucalypt stands were available. After the proposed modeling approach is described, results from the application to eucalypt (*E. globulus* Labill.) coppice stand management scheduling in Central Portugal are discussed.

## Stochastic DP Approach

### Model Building

The reader is referred to Kennedy (1986) and Hoganson et al. (2008) for an introduction to the use of DP concepts within a forestry framework and for a comprehensive review of DP applications to forest management scheduling.

The DP stochastic approach presented in this article aims to propose coppice stand optimal management policies, e.g., fuel treatment, stool thinning, and cycle lengths according to the stand state. It further aims to provide insight about the optimal number of cycles within a coppice system full rotation. This information is instrumental to address risk and uncertainty in an adaptive framework. For that purpose, our DP formulation breaks the coppice stand management problem into stages that are characterized by the cumulative number of harvests. Thus, the number of stages is determined by the maximum number of harvests over a whole coppice rotation.

This decomposition approach may be illustrated by the network corresponding to a deterministic problem, in which a state in each stage is characterized by the number of years since the stand was planted (Figure 1). Stand states (DP network nodes) at any stage thus depend on the range of cycle lengths. The DP arcs reflect management policies that may be implemented at any state (e.g., cycle length, stool thinning, and fuel treatment options). Thus, every arc encompasses a harvest decision. The backward recursion process may be used to solve this deterministic problem. An estimate of the soil expectation value (SEV) at the end of the rotation is needed to trigger the solution process. This estimate is associated with the “bare land nodes.” The values of these nodes thus correspond to the value of a net present value of an infinite repetition of rotations after the first. All nodes are connected to a bare land node if a clearcut may occur at the end of the first cycle (Figure 1).

Just as in the deterministic problem, at the beginning of each stage of the stochastic model, the stand state (DP network node) is characterized by one variable  $T_n$ , which corresponds to the time elapsed between planting and the harvest at the end of the  $(n - 1)$ th stage. Each DP arc is associated with a vector  $(I_n, V_n, M_n)$  that represents the set of management policies that may be implemented at the beginning of the  $n$ th stage.  $I_n$  stands for the number of years of the  $n$ th cycle, with  $I_n \in \Psi_n$ ;  $\Psi_n$  is the set of feasible cycle lengths.  $V_n$  corresponds to the average number of sprouts per stool after a stool thinning in the  $n$ th cycle, with  $V_n \in \Theta_n$ ;  $\Theta_n$  is the set of feasible average numbers of sprouts per stool. Finally,  $M_n$  corresponds to the number of fuel treatments over the  $n$ th cycle, with  $M_n \in \Pi_n$ ;  $\Pi_n$  is the set of feasible numbers of fuel treatments (Figure 2).

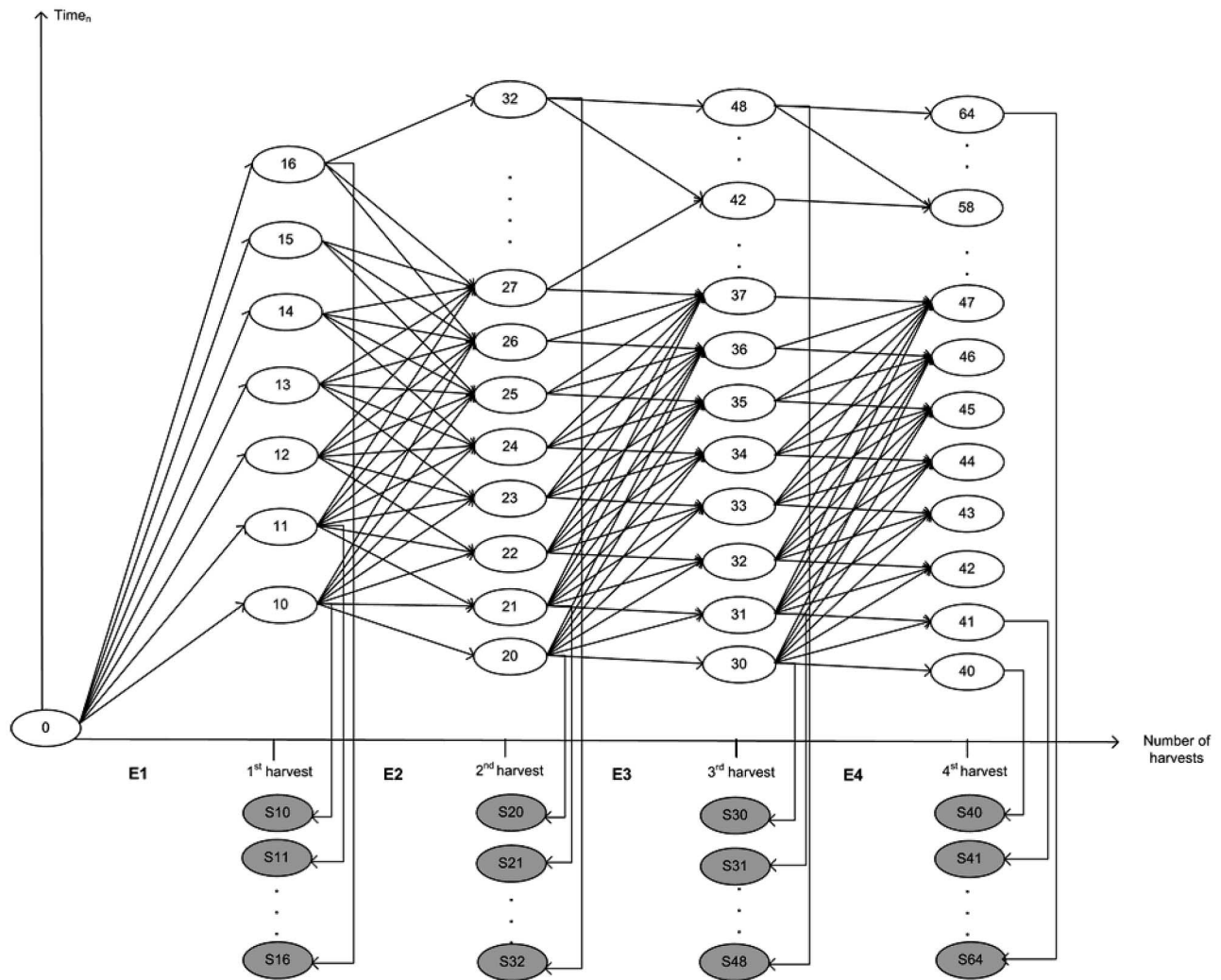


Figure 1. DP network design for deterministic coppice system management problem. DP network nodes above the horizontal axis represent the possible states for each stage and translate time since the stand was planted; nodes below the horizontal axis correspond to the called “bare land nodes,” nodes associated with clearcuts or destruction caused by a catastrophic event.

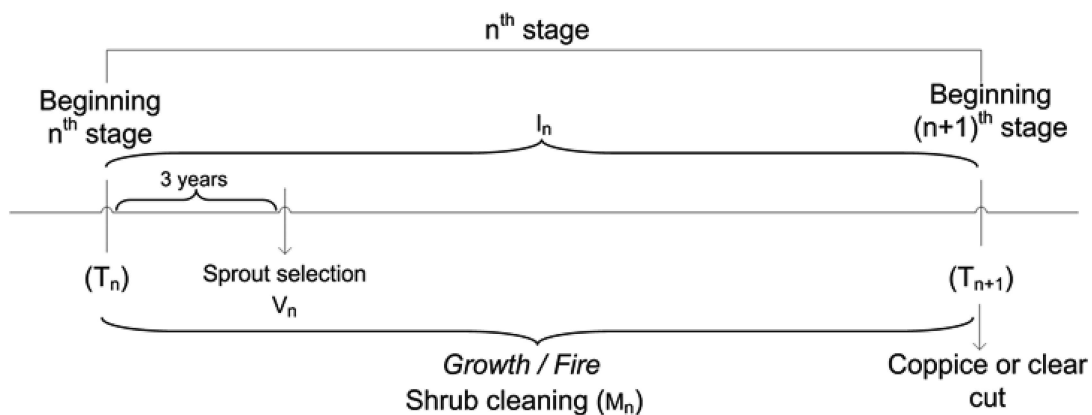


Figure 2. Characterization of the  $n$ th stage.

Backward recursive equations are used to solve the stochastic problem because they provide the information needed to implement DP within an adaptive framework. To start the backward recursion process, an estimate of the bare land value at the end of the rotation is associated with all harvest ages. This value is provided by a function  $S$ . The DP

return function computes the discounted net return of a set of management policies over a full cycle. The recursive function encompasses two subfunctions:  $G$  and  $F$ . The former considers catastrophic occurrence and probabilities of damage scenarios. It computes the expected values of the sums of net returns of management policies that may be

implemented at each state  $T_n$ , at the beginning of the  $n$ th stage with the net return associated with the optimal management policy to be implemented at either the state  $T_{n+1}$ , if no complete stand destruction occurs, or with the estimate of the bare land value associated with the age when the wildfire occurs, which is provided by the function  $S$ . The subfunction  $F$  selects the optimal path out of a node  $T_n$ , at the beginning of the  $n$ th stage.  $F_n(T_n)$  thus identifies the optimal management policy when the stand is in state  $T_n$ . This policy encompasses a decision regarding whether to clearcut or to implement one further coppice cycle and decisions associated with this potential cycle (length, stool thinning and fuel treatment).

Scenarios are characterized by both the probability of catastrophe occurrence over time and the damage caused. If we let  $J = J^1 \cup J^2$  be the set of all possible scenarios, we may define the subset  $J^1$  as the one that includes all scenarios when the stand does not have to be regenerated (e.g., no catastrophic occurrence or no mortality as a consequence of a catastrophic event).  $J^2$  becomes the subset of scenarios when the damage caused by the catastrophe forces the regeneration of the stand. The length of a cycle is thus affected by the introduction of catastrophe risk. Whereas the manager may plan a cycle with length  $I_n$ , the actual length will depend on the catastrophe scenario. The cycle length in the  $j$ th scenario ( $I_n^j$ ) may be defined by

$$I_n^j = \begin{cases} I_n, & \text{if } j \in J^1 \\ H^j, & \text{if } j \in J^2 \end{cases}$$

### Mathematical Description of the Stochastic Model

The model was defined as follows:

$$\text{Determine } Z = F_1(0) - CP \quad (1)$$

$$F_n(T_n) = \max_{\substack{I_n \in \Psi_n \\ V_n \in \Theta_n \\ M_n \in \Pi_n}} \{G_n(T_n, I_n, V_n, M_n), S_n(T_n)\} \quad n = 1, \dots, N \quad (2)$$

$$F_{N+1}(T_{N+1}) = S_{N+1}(T_{N+1}) \quad (3)$$

$$G_n(T_n, I_n, V_n, M_n) = \sum_{j \in J^1} p^j(T_n, I_n, V_n, M_n) [B_n^j(T_n, I_n, V_n) - CV_n^j(T_n) - L_n^j(T_n, I_n, M_n) + F_{n+1}(T_n + I_n)] + \sum_{j \in J^2} p^j(T_n, I_n, V_n, M_n) [B_n^j(T_n, I_n, V_n) - CV_n^j(T_n) - L_n^j(T_n, H^j, M_n) + S_{n+1}(T_n + H^j)] \quad n = 1, \dots, N \quad (4)$$

$$T_{n+1}^j = T_n + I_n^j, \text{ with } n = 1, \dots, N, j \in J^1 \cup J^2 \text{ e } T_1 = 0 \quad (5)$$

$$S_n(T_n) = \frac{F_1(0) - CR}{(1+i)^{T_n}}, \text{ with } n = 2, \dots, N+1 \text{ and } S_1(T_1) = 0 \quad (6)$$

where  $N$  is the maximum number of harvests over a coppicing system rotation;  $n = 1, \dots, N$  identifies the stage, defined by the number of harvests completed; the first harvest occurs at the end of stage 1;  $N+1$  identifies the end of stage  $N$ ;  $T_n$  is the number of years since the stand was planted at the beginning of stage  $n$ , it corresponds to the value of state variables defining a DP network node, at the beginning of the  $n$ th stage;  $T_{N+1}$  identifies the number of years since the stand was planted at the end of stage  $N$  when the stand is harvested the  $N$ th time;  $I_n$  and  $M_n$  are decisions regarding cycle length and fuel treatment scheduling over a cycle, respectively, as described under Model Building, with  $n = 1, \dots, N$ ;  $V_n$  is thinning option over a coppice cycle, as described under Model Building, with  $n = 2, \dots, N$ ;  $J^1$  and  $J^2$  are subsets of catastrophic scenarios that either do not or do force the stand to be regenerated, respectively, as described under Model Building;  $H^j$  is year when the scenario  $j \in J^2$  occurs, as defined under Model Building;  $Z$  is soil expectation value associated with the optimal management policy;  $F_n(T_n)$  is the optimal value of network node  $T_n$ , at the beginning of the  $n$ th stage;  $G_n(T_n, I_n, V_n, M_n)$  is the expected value of network path out of node  $T_n$  if the management policy involving decisions  $I_n$ ,  $V_n$ , and  $M_n$  is implemented at the beginning of the  $n$ th stage;  $S_n(T_n)$  is the value of the bare land node that is connected to network node  $T_n$ , at the beginning of the  $n$ th stage;  $B_n^j(T_n, I_n, V_n)$  is the discounted financial return associated with the sale of wood after a harvest or a catastrophe, at the  $n$ th stage, under the  $j$ th scenario;  $CV_n^j(T_n)$  is the discounted cost of a stool thinning option at the  $n$ th stage, under the  $j$ th scenario;  $L_n^j(T_n, I_n^j, M_n)$  is the discounted cost of a fuel treatment management scheduling option, at the  $n$ th stage, under the  $j$ th scenario;  $p^j(T_n, I_n, V_n, M_n)$  is the probability of occurrence of the  $j$ th catastrophe scenario, during the  $n$ th stage (it depends on the number of years since planting and on the management policy implemented at the beginning of the  $n$ th stage); CR is the conversion cost at the end of each rotation; and CP is the plantation cost at the beginning of the first rotation.

Equation 1 defines the coppice stand management objective of maximizing SEV. Maximum SEV is computed by the backward solution approach. Thus, it corresponds to the difference between the optimal value  $F_1(0)$  of the initial network node and the plantation cost. An estimate of the SEV is needed to satisfy the boundary condition and initiate the solution process because the value of  $F_1(0)$  is still unknown. This estimate is used to value all the bare land nodes. In the first iteration of the solution process,  $F_1(0)$ , in Equations 6, is replaced by an estimate, which is subtracted from the conversion cost and discounted the number of years since the planting (Equations 6). At the end of the solution process, the optimal value  $F_1(0)$  is compared with the estimate used. If these values are different, the iterative solution process continues using former  $F_1(0)$  to reestimate the land value (Hoganson et al. 2008). This method of successive approximations to the true SEV value can be proven to converge (Ferreira 2011).

Equations 2, 3, and 4 correspond to the DP recursive relations and determine the value of each node ( $T_n$ ), at the beginning of the  $n$ th stage, i.e.,  $F_n(T_n)$ . The value of the  $F$

function (Equations 2) depends on the value of the  $G$  function (Equations 4), which is an expected value set by catastrophe scenarios probabilities,  $p^j(T_n, I_n, V_n, M_n)$ , as described under Model Building. Equations 3 provide the values of the  $F$  function at the end of the  $n$ th stage.

Equations 5 correspond to the transition function and reflect the relationship between states of consecutive stages. The number of years from the planting to the harvest of the stand at the end of stage  $n$  corresponds to the sum of the number of years from the planting to the harvest at the end of stage  $n - 1$  with the duration of the  $n$ th cycle.

For simplicity it is assumed that only one catastrophe may occur over a cycle. This is often the case when the system encompasses a fast-growing species as cycles are short.  $H^j$  represents the year in the cycle when the catastrophe occurs. The model may be easily updated to consider scenarios when more than one catastrophe may occur over a cycle. It will be further assumed that if the catastrophe leads to any mortality, the stand must be regenerated.

The DP return function component (Equations 7), which computes the returns from the sale of timber, has thus two subcomponents. The first (Equations 8) provides the discounted return associated with the sale of live trees timber when harvest scheduling and stool thinning policies  $I_n$  and  $V_n$  are implemented, at the  $n$ th stage, if the  $j$ th scenario occurs. The second subcomponent (Equations 9) provides the discounted return resulting from the sale of timber from dead trees when those policies are implemented during the  $n$ th stage, if the  $j$ th scenario occurs.

$$B_n^j(T_n, I_n, V_n) = R_n^j(T_n, I_n, V_n) + Sal_n^j(T_n, V_n) \quad (7)$$

$$R_n^j(T_n, I_n, V_n) = \begin{cases} \frac{P1}{(1+i)^{T_n+I_n}} Vol(n, I_n, V_n) & , j \in J^1 \\ \frac{P1}{(1+i)^{T_n+H^j}} (1 - pm^j) Vol(n, H^j, V_n) & , j \in J^2 \end{cases} \quad (8)$$

$$Sal_n^j(T_n, V_n) = \begin{cases} 0 & , j \in J^1 \\ \frac{P2}{(1+i)^{T_n+H^j}} pm^j Vol(n, H^j, V_n) & , j \in J^2 \end{cases} \quad (9)$$

In Equations 8,  $P1$  represents the live trees stumpage price ( $\text{€}/m^3$ ) that is discounted  $T_n + I_n$  or  $T_n + H^j$  years if  $j$  belongs to  $J^1$  or  $J^2$ , respectively, the volume harvested ( $Vol(n, I_n, V_n)$  or  $Vol(n, H^j, V_n)$ ) is estimated by a growth-and-yield model, and  $pm^j$  stands for the proportion of trees that die as a consequence of the catastrophe.

In Equations 9,  $P2$  is the salvage price of dead trees timber ( $\text{€}/m^3$ ) that is discounted  $T_n + H^j$ ; the salvage volume in scenario  $j$  ( $Vol(n, H^j, V_n)$ ) is also estimated by a growth-and-yield model. If no mortality occurs, the return associated with the sale of salvageable timber is null.

The DP return function includes the cost of stool thinning in coppice cycles over the rotation. This cost thus occurs only from the second cycle on, i.e., for  $n > 1$ . It is the product of cost of sprout selection ( $CV$ ) by the number of sprouts  $NV_n$

(Equations 10). This value is discounted  $T_n + x$  years,  $x$  being the year in the cycle when the thinning takes place. It is assumed that it occurs only once over a coppice cycle.

$$CV_n^j(T_n) = \begin{cases} \frac{CV}{(1+i)^{T_n+x}} NV_n, & \text{if } I_n^j > x \\ 0, & \text{otherwise} \end{cases} \quad (10)$$

Finally, the DP return function includes the cost of fuel treatments. The frequency of fuel treatments in the  $n$ th cycle is given by  $\lfloor I_n/M_n \rfloor$ . Catastrophe occurrence will affect the number and timing of fuel treatments actually performed over the cycle. The information needed to assess this impact is provided by  $H^j$ , the year when the catastrophe occurs under scenario  $j$ . The binary variable  $PM_n(l, I_n^j, M_n)$  (Equations 11) indicates whether a fuel treatment occurs in year  $l$  of the  $n$ th cycle.

$$PM_n(l, I_n^j, M_n) = \begin{cases} 1, & \text{if a fuel treatment occurs in the } l^{\text{th}} \text{ year of the } n^{\text{th}} \text{ cycle} \\ & \text{under the } j^{\text{th}} \text{ scenario, with } l = 1, \dots, I_n^j \\ 0, & \text{otherwise} \end{cases} \quad (11)$$

Thus, in the case of scenarios when a catastrophic event does occur, the number and the timing of fuel treatments may be estimated by the procedure defined below. It will be assumed that if  $l = H^j$ , i.e., if a catastrophe occurs in year  $l$  of the  $n$ th cycle under scenario  $j$ , the understory biomass is destroyed, and thus there is no need to treat fuel in year  $l$  ( $PM_n(l, I_n^j, M_n) = 0$ ). Over the years  $l < H^j$ , the fuel treatments will occur as scheduled (Equations 12).

$$PM_n(l, I_n^j, M_n) = \begin{cases} 1, & \text{if } l = \lfloor r \frac{I_n}{M_n} \rfloor, \text{ with } r = 1, \dots, M_n \\ 0, & \text{otherwise} \end{cases} \quad (12)$$

After a catastrophic event occurs, i.e., in years  $l$  such that  $l > H^j$ , it is necessary to check the number of years from that occurrence up to the end of the cycle, to define the fuel treatment schedule. Thus, there are three possible cases:

1. If a catastrophic event leads to mortality, then the stand is clearcut and no more fuel treatments occur (if  $j \in J^2$  then  $PM_n(l, I_n^j, M_n) = 0$ );
2. If a catastrophic event does not lead to mortality (if  $j \in J^1 \setminus \{0\}$ ), then
  - a. if the number of years from the catastrophic event occurrence until the end of the  $n$ th cycle, is lower than the frequency defined for fuel treatments, no further treatments will be scheduled up to the end of  $n$ th stage. That is, if  $I_n - H^j < \lfloor I_n/M_n \rfloor$  then:

$$PM_n(l, I_n^j, M_n) = \begin{cases} 0, & \text{if } H^j < l < I_n \\ 1, & \text{if } l = I_n \end{cases} \quad (13)$$

- b. if the number of years from the catastrophic event occurrence until the end of the  $n$ th cycle is larger

than the frequency defined for fuel treatments, then the first treatment formerly scheduled to occur after the catastrophic event will not take place. Let  $l_0$  be the first year that  $l \leq l_0 < I_n$ , where a fuel treatment is planned, i.e.,  $l_0 = \lfloor r(I_n/M_n) \rfloor$ , for some  $r$  in  $\{1, \dots, M_n\}$ . Then

$$PM_n(l, I_n^j, M_n) = \begin{cases} 0, & \text{if } l = l_0 \\ 1, & \text{if } l > l_0 \text{ and } l = \left\lfloor r \frac{I_n}{M_n} \right\rfloor, \text{ with } r = 1, \dots, M_n \end{cases} \quad (14)$$

The cost of fuel treatments, in the  $n$ th cycle, depends on the actual length of the cycle,  $I_n^j$ , and it is calculated according to Equations 15:

$$L_n^j(T_n, I_n^j, M_n) = \sum_{l=1}^{I_n^j} \frac{L}{(1+i)^{T_n+l}} PM_n(l, I_n^j, M_n) \quad (15)$$

The backward recursion process provides information about the optimal management policy (cycle length, stool thinning, and fuel treatment schedule) to implement in any situation.

## Case Study

Eucalypt stands extend over 20% of total forest area in Portugal and provide key raw material for the pulp and paper industry. These stands are managed as a coppicing system, and eucalyptus is a species that is vulnerable to wildfire because it is highly flammable. To test the proposed stochastic approach, we will thus consider the optimization of eucalypt stand management scheduling under wildfire risk.

A typical eucalypt rotation may include up to two or three coppice cuts, each coppice cut being followed by a stool thinning in year 3 of the coppice cycle that may leave an average number of sprouts per stool ranging from one to two. Harvest ages range from 10 to 16. During the rotation, several shrub cleanings are performed (i.e., one to three fuel treatments per cycle). Thus, several costs have to be considered in the eucalypt stand management problem. The planting cost includes a fixed and a variable component and occurs only once in the beginning of the planning horizon. The former includes the soil preparation and the cleaning of existing shrubs costs. The latter includes the number of plants. At the end of a full rotation, when the stand has to be regenerated, there is a conversion cost that encompasses the cost of the destruction of old stools and the cost of planting new plants. Stool thinning costs depend on the existing number of sprouts and it occurs 3 years into the coppice cycle. The cost of each fuel treatment is fixed; therefore, the cost associated with the fuel treatment schedule only depends on the number of cleanings that will be performed over each cycle.

Wildfires have an impact on eucalypt stand management scheduling. Typically, if mortality occurs, the stand is regenerated whatever the mortality rate. Dead tree timber is

sold at a salvage price that is about 75% of the original price. Severe wildfires thus have a substantial impact on the management schedule and both its revenues and its costs. Model solving considered average eucalypt pulpwood prices and operations costs in Portugal (Tables 1 and 2). A 4% rate was used to discount costs and revenues.

## The Stochastic DP Model

Stages are characterized by the number of harvests completed over one rotation. The DP network encompasses four stages corresponding to four cycles because the maximum number of harvests over one rotation is four ( $N = 4$ ). The states in any stage represent the number of years since the stand was planted. The end of the fourth stage, when the fourth harvest may take place, is represented by  $N + 1 = 5$ .

The design of the DP network took into account all management options over a cycle. In each cycle, the harvest age may range from 10 to 16 years. The set  $\Psi_n$  includes feasible values of variable  $I_n$ , the duration of the  $n$ th cycle in the  $n$ th stage (Table 3). In each coppice cycle, stool thinning in year 3 may leave on average 1, 1.5, or 2 sprouts per stool ( $V_n$  values in set  $\Theta_n$ ) (Table 3). The variable  $M_n$  represents the number of fuel treatments during the  $n$ th cycle. When a harvest occurs, shrubs are also removed. Two further treatments may be scheduled over a cycle. Thus, the value of this variable may range from 1 to 3 in the set  $\Pi_n$  (Table 3).

If the model was deterministic, the values of state variable  $T_n$  would extend from 0 at the beginning of stage 1 to 64 at the end of stage 4 (Table 4). In the stochastic model, because some wildfires lead to mortality and to the need for regenerating the stand at different ages, there are other possible states in the DP network. These states are characterized by the number of years from planting up to the year when a wildfire occurrence leads to a clearcut (Table 4).

The number of trees planted per hectare, at the beginning of the rotation, is a parameter defined before initialization of the solution process that depends on the spacing adopted. For testing purposes, we considered three possible values for the number of trees planted per hectare (NPL): 1,111, 1,250 and 1,667.

## Vegetation Growth Models

Eucalypt growth was estimated using the stand-level growth-and-yield model Globulus 3.0 (Tomé et al. 2006). This model was developed for Portuguese eucalypt stands. After each coppice harvest, it is necessary to calculate the number of live stools at the beginning of the next cycle, because there is a percentage of stools that die in the

**Table 1. Timber stumpage and salvage prices used for eucalypt stand management scheduling.**

	Prices (€/m <sup>3</sup> )
Eucalypt pulpwood stumpage price	36
Eucalypt pulpwood salvage price	27

Source: Personal communication by forest managers in the Portuguese forest industry.

**Table 2. Operational costs.**

	Fixed cost (€/ha)	Variable cost
Shrub removal cost	167	
Plantation cost	725	0.14€ × no. of plants
Stool thinning cost		0.15€ × no. of sprouts
Conversion cost	1204	0.14€ × no. of plants

Source: Comissão de Acompanhamento de Operações Florestais Database of ANEFA (The National Association of Forest, Agriculture and Environment Enterprises).

**Table 3. Possible values for management decisions.**

Silviculture parameters	Set of feasible values
Harvest age	$\Psi_n = \{10, 11, \dots, 16\}$
Average no. of sprouts per stool	$\Theta_n = \{1; 1.5; 2\}$
No. of treatments over a cycle	$\Pi_n = \{1, 2, 3\}$

**Table 4. Possible states for deterministic and stochastic models.**

Stage	Deterministic states	Stochastic states
1	$T = \{0\}$	$T = \{0\}$
2	$T = \{10, 11, \dots, 16\}$	$T = \{1, 2, \dots, 16\}$
3	$T = \{20, 21, \dots, 32\}$	$T = \{11, 12, \dots, 32\}$
4	$T = \{30, 31, \dots, 48\}$	$T = \{21, 22, \dots, 48\}$
5	$T = \{40, 41, \dots, 64\}$	$T = \{31, 32, \dots, 64\}$

transition between cycles. In this study, the rate of mortality considered for stools after each coppice harvest was 20%.

Understory growth was estimated according to a model developed by Botequim et al. (2009). According to this model, the understory biomass level depends on its age and on the basal area of the eucalypt stand, and its growth is simulated by the following equation:

$$Biom = 17.745 (1 - \exp^{-(0.085 \text{ understory age} + 0.004AB)}) \quad (16)$$

It is assumed that, if there is a fuel treatment or a wildfire, the understory biomass level becomes null.

### Wildfire Occurrence and Damage Models

Wildfire is a stochastic element. Thus, the state of the stand cannot be projected into the future with certainty. Nevertheless, it may be predicted using wildfire occurrence and damage scenarios probabilities. These scenarios were built according to recent research of wildfire occurrence and damage models in eucalypt stands in Portugal (Botequim et al. 2011, Marques et al. 2011a, 2011b). Botequim et al. (2011) used a binary logistic regression approach to develop a model that provides annual wildfire risk probability in even-aged eucalypt stands in Portugal (Equation 17).

$$Pf_a = \frac{1}{1 + e^{-(-2.4368 + 0.0815Biom - 0.0671a - 0.0671AspSW + 0.000613N + 0.0373Dg)}} \quad (17)$$

where  $Pf_a$  is the probability of wildfire occurrence in the stand,  $Biom$  is the understory biomass (tons per ha),  $a$  is the age of the stand (years),  $AspSW$  is southwest aspect,  $N$  is the number of trees (per ha), and  $Dg$  is the quadratic mean diameter (cm). For testing purposes, the parameter  $AspSW$  was assumed to be equal to 0. Marques et al. (2011b) also used logistic regression methods to develop postfire mortality models in eucalypt stands in Portugal. In this research it was observed that mortality took place in 44 of the 92 stands crossed by fire. The proportion observed was used to predict whether mortality will occur in a stand after a wildfire ( $P_{mort} = 44/92$ ). To measure the level of damage (e.g., proportion of dead trees in the stand) if mortality occurs, Marques et al. (2011b) developed the following model (Equation 18):

$$P_{am} = \frac{1}{1 + e^{-(0.8537 + 0.00244Alt + 0.0197Slope - 0.0851AB + 0.2246Sd)}} \quad (18)$$

where  $p_{am}$  is the proportion of dead trees,  $Alt$  is altitude (m),  $Slope$  is measured in degrees,  $AB$  is the basal area (m<sup>2</sup>/ha) and  $Sd$  is the SD of the diameter of trees (cm). For testing purposes,  $Alt$ ,  $Slope$ , and  $Sd$  were assigned the values 50, 0, and 4, respectively.

Wildfire risk was thus incorporated into the stochastic model by defining wildfire occurrence and damage scenarios according to the models developed by Marques et al. (2011b) and Botequim et al. (2011). Wildfire occurrence probability increases with the number of trees per ha, the understory biomass, and the tree quadratic diameter. Whenever mortality occurs, the damage increases with the stand basal area.

In this research it was assumed that a wildfire may occur only once over a cycle and that annual occurrence probabilities are independent of each other. Wildfire recurrence periods do tend to be larger than 7 years in eucalypt stands in Portugal. Thus, the definition of wildfire scenarios at each stage considered the probability of one wildfire occurrence over the cycle (Equation 19),

$$Pi_a = \begin{cases} Pf_a \prod_{q=1}^{a-1} (1 - Pf_q), & \text{if } a > 1 \\ Pf_a, & \text{if } a = 1 \end{cases} \quad (19)$$

where  $Pf_a$  is the probability of wildfire occurrence in a stand that is  $a$  years old and  $Pi_a$  is the probability of wildfire occurrence in year  $a$  of the cycle.

Equations 20–22 estimate the probabilities associated with each scenario,  $p^j$ :

$$p^j = Pi_j(1 - P_{mort}), \text{ if } j \in J^1 \setminus \{0\} \quad (20)$$

$$p^j = Pi_j P_{mort}, \text{ if } j \in J^2 \quad (21)$$

$$p^j = 1 - \sum_{j \in J^1 \setminus \{0\} \cup J^2} p^j, \text{ if } j = 0 \quad (22)$$

The proportion of dead trees as a result of the  $j$ th wildfire occurrence scenario, during the  $n$ th stage,  $pm^j$ , is null for every  $j \in J^1$  (i.e., scenarios without mortality) and assumes

the value  $p_{am}$  for scenarios  $j \in J^2$  (i.e., scenarios with mortality).

If a wildfire occurs, the understory biomass becomes null, thus affecting the fuel treatment schedule. No fuel treatment is needed immediately after a wildfire.

In summary, in each stage, two sets of scenarios are considered (Figure 3). The first is designated by  $J^1$ , and it includes the scenario of no wildfire occurrence over the cycle ( $j = 0$ ) and the full range of scenarios involving the occurrence of moderate wildfires that do not cause mortality ( $j = 1, \dots, I_n$ ). Thus,  $J^1 = \{0, 1, 2, \dots, I_n\}$ . The second set is denoted by  $J^2$  and includes all scenarios involving the occurrence of severe wildfires that lead to mortality,  $J^2 = \{I_n + 1, I_n + 2, \dots, I_n + I_n\}$ . The

probabilities of each scenario within the sets  $J^1$  and  $J^2$  decrease with the time since planting or since the coppice harvest in the case of the first and the remaining cycles, respectively (Table 5).

## Results

The DP algorithm was programmed with C++, and the test problem was solved with a desktop computer (CPU Duo P8400 with 3GB of RAM).

First, the problem was solved as if no wildfire risk existed to assess the difference between the management planning proposal by a model that takes into account risk

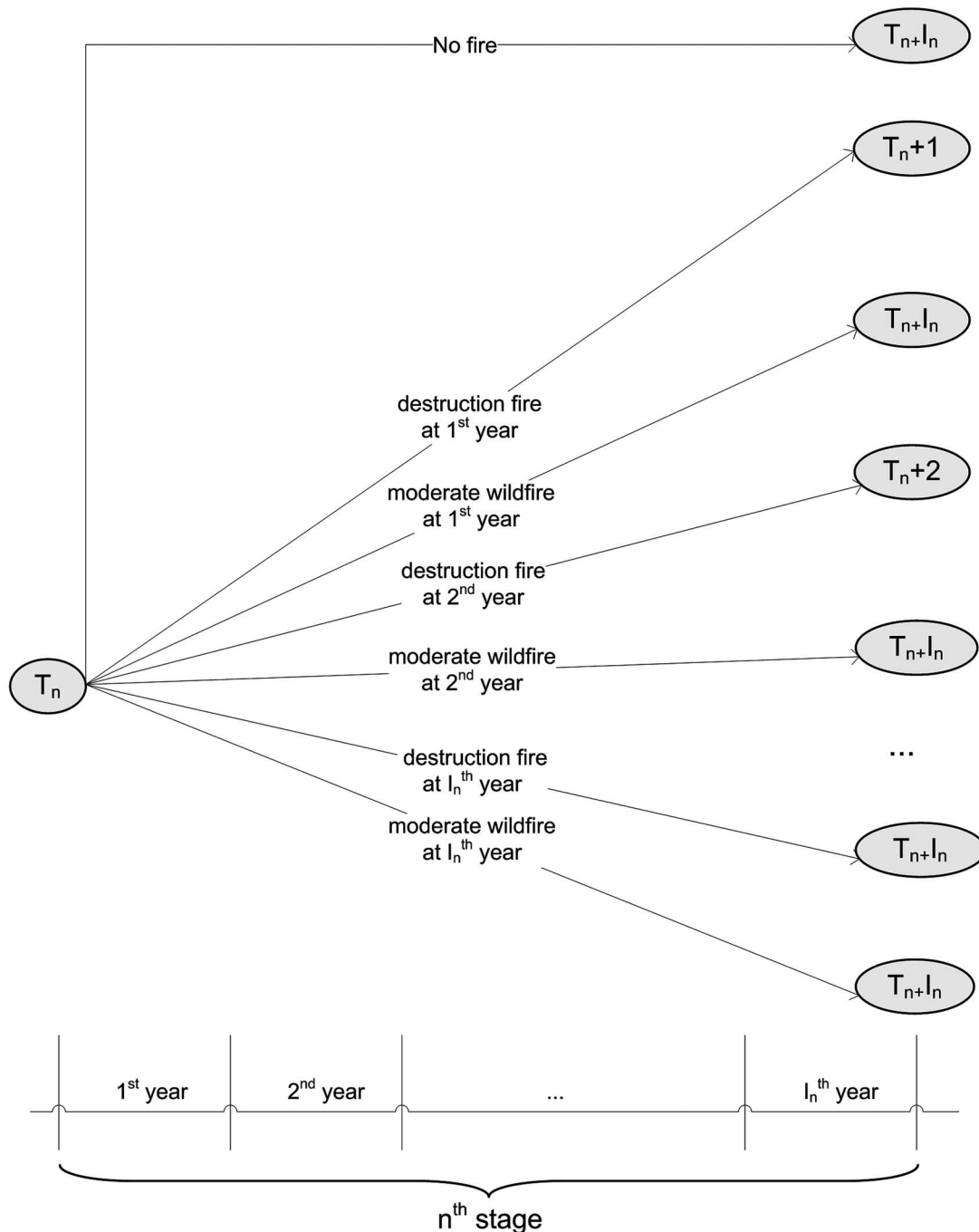


Figure 3. Wildfire occurrence and damage scenarios for the  $n^{\text{th}}$  stage.

**Table 5. Wildfire scenario probabilities, for  $j \in J^1$ , when  $I_n = 16$ ,  $M_n = 1$ , and  $V_n = 2$ .**

Scenarios ( $j \in J^1$ )	Year of wildfire occurrence	Probabilities			
		1st cycle	2nd cycle	3rd cycle	4th cycle
0		0.018514	0.011045	0.025178	0.037218
1	1	0.084153	0.098858	0.077672	0.066007
2	2	0.078186	0.087546	0.073169	0.064621
3	3	0.069655	0.074466	0.066261	0.060772
4	4	0.059807	0.061182	0.058041	0.055247
5	5	0.049812	0.048808	0.049393	0.048765
6	6	0.04047	0.038019	0.041043	0.041997
7	7	0.032226	0.029056	0.033445	0.035435
8	8	0.025253	0.021878	0.026825	0.029396
9	9	0.01954	0.016287	0.021244	0.024048
10	10	0.014976	0.012025	0.016659	0.019453
11	11	0.011398	0.008828	0.012967	0.015597
12	12	0.008634	0.006461	0.01004	0.01242
13	13	0.006522	0.004722	0.007746	0.009841
14	14	0.004922	0.003453	0.005966	0.007772
15	15	0.003715	0.00253	0.004593	0.006125
16	16	0.002809	0.001859	0.003539	0.004824

and the proposal by a model that does not. In the deterministic case, only one scenario exists ( $j = 0$ ); the corresponding probability is  $p^0 = 1$ , and the proportion of dead trees is null ( $pm^0 = 0$ ). Optimal SEV increased with the number of planted trees and ranged from 4,390€/ha if the number of trees planted is 1,111 to 5,153€/ha if that number is 1,667 (Table 6). The former corresponded to a 63-year optimal rotation that encompassed four cycles, with the first cycle extended over 15 years and the remaining cycles extended over 16 years. No fuel treatments were scheduled other than the ones that are compulsory when the stand was harvested. This is understandable, because in the deterministic case, fuel treatments would increase costs without any additional benefit. The optimal prescription proposed by the deterministic model encompassed stool thinning options that left an average of 2 sprouts per stool in all three coppice cycles.

We used the optimal SEV estimate by the deterministic model to trigger the backward recursion process to solve the stochastic problem. If NPL was 1,111, the initial estimate of  $F_1(0)$  was thus 4,390€/ha. The iterative solution process runs until a stopping condition is satisfied. For our testing purposes, the solution process stopped when the difference between the optimal SEV and the estimate of  $F_1(0)$ , used to start the process, was lower than 0.01. In this case, convergence took 20 iterations and approximately 17 seconds. The optimal SEV is 2,382.72€/ha (Table 7). If either no wildfire occurs or wildfires do not cause mortality, the optimal rotation will extend to 64 years and will encompass four cycles of 16 years each. No fuel treatments were scheduled

other than the ones that are compulsory when the stand is harvested. Mild wildfires have the same effect as a fuel treatment. Just as in the deterministic case, the optimal prescription encompassed stool-thinning options that left an average of two sprouts per stool in all three coppice cycles. The present value of expected net income in the first cycle was 1,661€/ha. If the stand survives up to the beginning of the second cycle and if the first coppice cut occurs at 16 years of age, the present value of expected net income in the second cycle is 780€/ha. The expected net returns in the third and the fourth cycles are 381 and 187€/ha, if the stand survives up to 48 and 64 years, respectively. The expected present value of future rotations is 155€/ha.

Because of the wildfire occurrence and damage probabilities scenarios, the expected length of all cycles is much lower than what is proposed by both the deterministic and the stochastic model as the optimal management policy ( $I_n$ ) (Table 8). The expected SEV associated with the optimal management policy, proposed by the stochastic model (Table 7), depends on wildfire occurrence and damage probabilities. As a consequence, these values are lower than the ones obtained by the deterministic model; SEV decreases with wildfire risk as expected. Yet both the number of cycles and the cycle length remain about the same in the solutions by both the deterministic and the stochastic models. The occurrence of a wildfire in any given year of a cycle was assumed to depend on the nonoccurrence of a wildfire in earlier years of the cycle. The probability of a catastrophe occurring later in the cycle is thus set to decrease (Table 5). Accordingly, the value of the recursive function  $G_n$  may

**Table 6. Results for deterministic case.**

NPL	1st cycle			2nd cycle			3rd cycle			4th cycle			Full rotation		
	$I_n$	$M_n$	$V_n$	$I_n$	$M_n$	$V_n$	$I_n$	$M_n$	$V_n$	$I_n$	$M_n$	$V_n$	Yr	Fuel treatment	SEV (€/ha)
1,111	15	1		16	1	2	16	1	2	16	1	2	63	4	4,390.12
1,250	15	1		16	1	2	16	1	2	16	1	2	63	4	4,584.98
1,667	14	1		14	1	2	16	1	2	14	1	2	58	4	5,153.99

**Table 7. Results for stochastic model.**

NPL	1st cycle			2nd cycle			3rd cycle			4th cycle			Full rotation		
	$I_n$	$M_n$	$V_n$	$I_n$	$M_n$	$V_n$	$I_n$	$M_n$	$V_n$	$I_n$	$M_n$	$V_n$	Yr	Fuel treatment	SEV (€/ha)
1,111	16	1		16	1	2	16	1	2	16	1	2	64	4	2,382.72
1,250	16	1		16	1	2	16	1	2	16	1	2	64	4	2,494.90
1,667	16	1		16	1	2	16	1	2	16	1	2	64	4	2,811.64

**Table 8. Expected rotation length for SDP model with NPL = 1,111.**

	Expected rotation length
1st rotation	10.71
2nd rotation	10.46
3rd rotation	10.86
4th rotation	11.12

increase with  $I_n$ , as the stand value growth rate may offset the increase of the probability of occurrence of a catastrophe with the cycle length and a risk premium that becomes lower with the cycle year. Nevertheless, if all scenarios are assigned the same probability, the stochastic model proposes cycle lengths that are lower than the ones proposed by the deterministic approach (Table 9), as expected.

When wildfire risk is considered, the stand may have to be harvested earlier than proposed by the model. This is the case when the wildfire leads to tree mortality. In all other scenarios, the management options may be implemented as proposed by the stochastic approach solution. Nevertheless, the introduction of wildfire risk provides the expected value of the optimal SEV. It further provides information about the optimal policy to implement over a cycle.

With the purpose of monitoring the response of the model to changes in several parameters, a sensitivity analysis was performed. Variations in the discount rate and in prices were tested to assess their impact on the SEV.

The soil expectation value decreases significantly with the discount rate, as expected. The opportunity cost is bigger, and the losses associated with the investment in the stand for timber production are higher. The discount rate increase has an impact similar to the introduction of a wildfire risk premium and may lead to a reduction in rotation length (Table 10).

On the other hand, a stumpage price decrease of up to 20% does not have an impact on the policies proposed by the stochastic model; i.e., the number of cycles, the cycle length, the number of fuel treatments, and the number of sprouts selected by stool remain unchanged (Table 11).

However, as expected, the soil expectation value does decrease.

Results follow the same trends in the case of the two other values of the initial number of planted trees (1,250 and 1,667).

## Discussion and Conclusions

In this research, risk was considered an endogenous factor in the model. It was assumed that wildfire occurrence and damage probabilities were affected by stand age, shrub biomass, and number of trees. This assumption was influential in fulfilling the research ultimate goal of proposing optimal management policies for coppice systems management planning under risk.

Dynamic programming is very useful for stand-level optimization because it helps avoid the problem of needing to enumerate and evaluate all possible management options (Hoganson et al. 2008). Other studies have introduced DP to optimize the length of each coppice cycle as well as the number of harvests within a coppice system rotation (Tait 1986, Díaz-Balteiro and Rodriguez 2006). However, none of these studies addressed the impact of catastrophic risk in management planning. The proposed stochastic DP solution approach did contribute to addressing wildfire occurrence and damage scenarios in short-rotation coppice systems management scheduling. It provides valuable information about the best management planning policies to address risk in any given state of a short-rotation coppice system.

The stochastic DP approach presented in this article proposes coppice stand optimal management policies, e.g., fuel treatment, stool thinning, and cycle lengths according to the stand state. It further provides insight about the optimal number of cycles within a coppice system full rotation. This information is instrumental in addressing risk and uncertainty in an adaptive framework. In this study, we present an application to a typical eucalypt stand in Central Portugal. However, the proposed approach may help integrate catastrophic risk in other short-rotation coppice systems.

In contrast to conventional deterministic DP approaches,

**Table 9. Results for the stochastic model when all wildfire occurrence scenarios are assigned the same probability (0.03).**

NPL	1st rotation			2nd rotation			3rd rotation			4th rotation			Full cycle		
	$I_n$	$M_n$	$V_n$	$I_n$	$M_n$	$V_n$	$I_n$	$M_n$	$V_n$	$I_n$	$M_n$	$V_n$	Yr	Fuel treatment	SEV (€/ha)
1,111	13	1		14	1	2	14	1	2	16	1	2	57	4	2,917.02
1,250	13	1		14	1	2	14	1	2	16	1	2	57	4	3,091.55
1,667	13	1		14	1	2	14	1	2	16	1	2	57	4	3,595.57

**Table 10. Sensitivity analysis for variations in discount rate.**

NPL	Discount rate (%)	1st cycle			2nd cycle			3rd cycle			4th cycle			Full rotation		
		$I_n$	$M_n$	$V_n$	$I_n$	$M_n$	$V_n$	$I_n$	$M_n$	$V_n$	$I_n$	$M_n$	$V_n$	Yr	Fuel treatment	SEV (€/ha)
1,111	2	16	1	-	16	1	2	16	1	2	16	1	2	64	4	7,298.49
	4	16	1	-	16	1	2	16	1	2	16	1	2	64	4	2,382.72
	6	15	1	-	16	1	2	16	1	2	16	1	2	63	4	790.42
	8	14	1	-	14	1	2	14	1	2	16	1	2	58	4	65.30

**Table 11. Sensitivity analysis for variations in prices.**

NPL	P1	P2	1st cycle			2nd cycle			3rd cycle			4th cycle			Full rotation		
			$I_n$	$M_n$	$V_n$	$I_n$	$M_n$	$V_n$	$I_n$	$M_n$	$V_n$	$I_n$	$M_n$	$V_n$	Yr	Fuel treatment	SEV (€/ha)
1,111	... €/m <sup>3</sup> ...		16	1	-	16	1	2	16	1	2	16	1	2	64	4	1,341.87
	28.8	21.6	16	1	-	16	1	2	16	1	2	16	1	2	64	4	2,382.72
	36	27	16	1	-	16	1	2	16	1	2	16	1	2	64	4	3,423.57

P1, timber stumpage price; P2, salvage price.

the solution by our formulation does not produce a pre-defined optimal prescription or “optimal path.” However, it produces optimal stand management policies according to the stand state at any time. This means that to check what management policy to implement if a change occurs as a consequence of a catastrophe, the forest manager just has to check his new stand state and, based on this information, determine the management policy that is best adapted to the new situation. This formulation of the problem and corresponding solution match with the definition of adaptive forest planning (Zhou et al. 2008).

The results presented show that the model is sensitive to discount rate changes. The number of stages and the lengths of the cycles are affected by changes in this parameter. A lower discount rate may lead to lower cycle length even if the number of coppice cycles remains constant. Moreover, as expected, the soil expectation values decrease with discount rate. The introduction of wildfire risk has a similar effect on the optimal soil expectation value. This is in accordance with other studies (e.g., Reed 1984, Díaz-Balteiro and Rodriguez 2008) that interpreted the impact of wildfire risk as a premium added to the discount rate in the case of a risk-free environment.

Results demonstrated the importance of the way probabilities are assigned to catastrophe scenarios. If the probability of a scenario occurring later in a cycle is dependent on the nonoccurrence of a catastrophe earlier, stand growth rates may overcome the risk marginal increase and lead to longer optimal cycles. Nevertheless, the expected cycle length by the stochastic model is always lower than the cycle length proposed by the deterministic model. Results further confirm the importance of fuel treatments when wildfire risk and damage are addressed. Soil expectation value increases when prescriptions do include understory fuel management. Further, cycle lengths may be longer regardless of the interest rate. Stands become less vulnerable to fire damage and therefore the optimal harvest age may increase. These results are concordant with some authors (e.g., Amacher et al. 2005, González-Olabarría et al.

2008, Pasalodos-Tato and Pukkala 2008, Garcia-Gonzalo et al. 2011).

The convergence of the stochastic model is quite good. Generally, a moderate number of iterations are needed to get convergence. Even if the SEV estimate is far from the correct value, the convergence process is straightforward and efficient.

One novelty of this article is that fuel biomass is included in the risk model and therefore fuel treatments modify wildfire risk. The characterization of shrub biomass in terms of the shrub’s age may be enhanced by the development of a better model describing the shrub biomass growth under tree cover. The proposed SDP may be easily updated to include such a model.

Forest management scheduling may be addressed at different spatial scales, namely stand-level management and landscape-level management. When catastrophic risk in coppice systems management scheduling models is addressed, it is important to consider both spatial scales. Further research is needed to examine risk at the landscape level. The proposed approach may provide valuable information about stand-level subproblems within the wider landscape-level master problem.

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