Use of available phasor measurements for system observability: a case study

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***Abstract*-This paper proposes the use of available PMU measurements for establishing network observability. State estimation is an important EMS application for providing real time, reliable and qualitative information on the system state. The observability analysis is the necessary function of the whole network state estimation.**

This paper addresses the incorporation of PMU measurements with the purpose of full network observability. The methodology is validated by the results on IEEE 14, 30 and 57 bus test system.

I. INTRODUCTION

In recent years, the electricity industry has undergone drastic changes due to a world wide deregulation process that has significantly affected power system management and energy markets. The belief is that competitive markets will lead to more efficient power generation, more technological innovations, and eventually to lower retail prices. In addition, the power system network is growing larger and more complex as there are many independent power generators adding generation to the electric system and a continuous increase of load demand. In this situation, the function of state estimation is becoming more important, because it is the main tool for monitoring and control based on the real-time data received from the measurement system. The economic dispatch, optimization applications, dynamic security analysis (including voltage and transient stability), and other applications strongly depend on the accuracy of data provided by the state estimator. Ensuring reliability on the estimated state is vital in terms of security concerns. Recently, synchronized phasor measurements have started to be available in the system. PMUs (Phasor Measurement Units) are devices equipped with Global Positioning System receivers. These receivers allow for the synchronization of the several readings taken at distant points. PMUs rely on a GPS time signal for extremely accurate time-sampling of the power system information. A GPS satellite receiver provides a precise timing pulse, which is correlated with sampled voltage and current inputs, usually the three phase voltages and the currents in lines [1]. There are several topics being discussed in the literature on the use of phasor measurements in state estimation. The incorporation of PMUs in state estimation is referred [2]-[6]. Methods for an optimal measurement placement of phasor measurement units for power system state estimation are addressed in [7]-[12]. Observability analysis is presented in [12]-[13].

As network observability must be determined prior to state estimation. Before performing a state estimation it is necessary to guarantee network observability.

In this paper, an approach for verifying system observability and reestablishing it by using PMU measurements in the presence of conventional measurements is presented.

An observability analysis is performed and if there are any unobservable parts of the network, then a procedure for using other measurements with the purpose of adding observable network region will be used. It makes use of measurements obtained from available PMU in the network. The goal is to make the entire network observable

II. MATHEMATICAL MODEL

A. *Observability analysis*

Observability analysis is a search process for portions of a power network for which, given the network and measurement topology, state estimation can be performed. It can be assessed by considering the topology of the network and the types and locations of the measurements. Two different observability concepts can be defined for the linear system model [Clements et al.]:

Numerical observability: it is defined as the ability of the system model to be solved for a state estimate.

Given an N-bus network, and m measurements of voltage and current phasors, the linear equations relating the measurements, and the state vector are:

$$z = Hx + e \quad (1)$$

where the vector z is linearly related to the n-dimensional state vector x containing N-bus phase angle state variables, Hx is decoupled jacobian matrix of real power measurements relating to the fase angles, e is the additive measurement error

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vector and the measurement error covariance matrix is assumed to be the identity matrix. The number of buses made observable is inferred by the rank of the matrix H (the gain matrix). If the matrix H is of full rank and well-conditioned, then the system is considered to be numerically observable; the number of buses with state variables that cannot be calculated is determined by

$$\frac{N-(Rank(H)+1)}{2} \tag{2}$$

When the system is completely observable, there are not unobservable buses, and the result of (2) is zero.

Topological observability: it is defined as the existence of at least one spanning measurement tree of full rank in the network. This tree connects all the nodes through branches with a metered or a calculated current phasor assigned to each of them. In addition, the associated measurement-to-node incidence submatrix has a rank equal to the number of the branches of that tree .

Thus the observability of a system can be decided through either Jacobian matrix analysis or topological analysis.

In this paper, an algorithm based on the observability analysis method introduced earlier in [14] is used for observability analysis. Let us briefly review the method first.

Decoupled gain matrix for the real power measurements can be formed as:

$$G_{pp} = H_{PP}^{T} H_{PP} \tag{3}$$

where,

$$H_{PP} = \frac{\partial h_P}{\partial \theta} \tag{4}$$

corresponds to decoupled jacobian matrix for real power measurements. Unobservable branches can be easily identified by the factorization of the gain matrix. Cholesky factorization of the gain matrix will be interrupted as soon as a zero pivot is found.

Consider the step where the first zero pivot is encountered during the factorization of the singular gain matrix, as shown below:

$$G_{pp} = \begin{bmatrix} d_1 & & & & & & & \\ & d_2 & & & & & & \\ & & \ddots & & & & & \\ & & & d_i & & & & \\ & & & & 0 & 0 & \cdots & 0 \\ & & & & 0 & \times & \times & \times \\ & & & & \vdots & \times & \times & \times \\ & & & & 0 & \times & \times & \times \end{bmatrix} \tag{5}$$

When a zero pivot is encountered, a 1.0 replaces it and the corresponding entry of the right hand side vector t_P will be assigned an arbitrary value. These arbitrary values should remain distinct from each other. One possible choice is integers in increasing order, such as 1, 2, 3, etc. Then, forming the following vector can identify the unobservable branches:

$$P_b = A_{inc} G_{PP}^{-1} t_P \tag{6}$$

here A_{inc} is the branch-bus incidence matrix.

If an element of P_b is nonzero, the corresponding branch will be unobservable. Removing the unobservable branches and injections incident on them, the procedure is repeated until no more branches are found unobservable.

Unobservable branches will separate observable islands. Identifying unobservable branches will lead to the identification of the observable islands. A system with no unobservable branches will be fully observable.

B. Formulation using PMU measurements

Current energy management systems (EMSs) need to accurately monitor in real time the power system state variables, i.e. the voltage phasors (voltages in module and phase) of each bus. The fundamental tool is the state estimation that constitutes the core of all the control operations. A minimum number of appropriately distributed PMUs is needed in order to carry out its state estimation.

In this section, the approach of using phasor measurements by adding them to the measurement set such that the network regains its observability is presented.

Once identified the observable regions of the network, other measures can be added in order to link all regions into one. The measurements to be added should be branch measurements from branches that link observable regions of the network, or injections on the buses that are in the frontier of the observable regions of the network.

The method used selects a set measurements that are able to render possible system observability without contaminating observable regions. These measurements will be formed after the measurements taken from available PMUs in the network. They will make observable all non observable branches, thus adding the various observable regions of the network into one full observable island.

These measurements can be defined as measurements obtained directly from PMUs.

PMUs can measure voltage and currents phasors, currents injections as well as current flows.

The phasor current from bus i to bus j, derived from π line model (Fig. 1), can be expressed as:

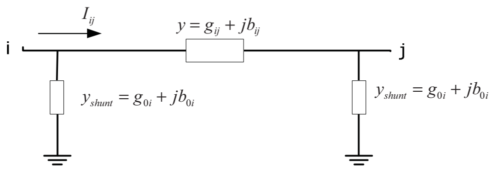

Fig. 1. π Line model

$$\overline{I_{ij}} = \overline{V_i}(g_{0i} + jb_{0i}) + (\overline{V_i} - \overline{V_j})(g_{ij} + jb_{ij}) \tag{7}$$

where $g_{ij}+jb_{ij}$ establishes the series admittance of the branch that connects nodes ij and $g_{0j}+jb_{0j}$ the shunt admittance of the branch that is connected to bus i.

The procedure which realizes complete observability follows the steps listed below:

1) Use of PMUs measurements available on the network. It's assumed that a PMU at a bus makes that bus and its neighbors observable.

2) Establish a measurements set which contain the measures candidates to link the observable regions of the network. These candidates are measures of unobservable branches and buses on the border of observable islands.

3) Judge observability of the system according to observability analysis.

4) If complete observability has not been achieved then the first steps should be preceded again for the unobservable region until complete observability is realized.

The network observability can be achieved according to what was stated before and it serves the purpose of having a measurement set that guarantees full network observability. It can precede other system analysis as state estimation.

III. APPLICATION EXAMPLES

The proposed approach is applied to IEEE 14 bus, 30 bus and 57 bus systems.

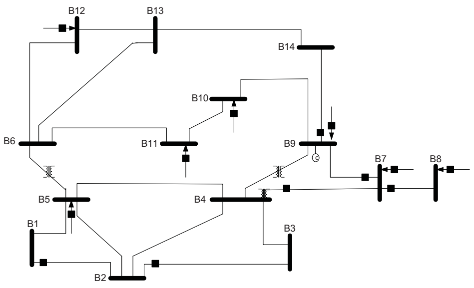

Fig. 1. Configuration set used on the IEEE 14 Bus test system

The configuration measurement for IEEE 14-bus test system is shown in Fig.1. The observability analysis is presented in detail for the IEEE 14 bus test system. Since measures of active and reactive power are available observability $P-\theta$ and $Q-V$ can be tested separately. Thus an $P-\theta$ observability analysis was made.

According to the measurement configuration (Fig.1) the Jacobian matrix of measurement can be characterize as:

$$H_{PP}=\begin{bmatrix}
-1 & -1 & 0 & -1 & 4 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & -1 & 0 & 0 & 3 & -1 & -1 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & -1 & 0 & 0 & -1 & 0 & 4 & -1 & 0 & 0 & 0 & -1\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 2 & -1 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & -1 & 2 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 2 & -1 & 0\\
1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & -1
\end{bmatrix} \tag{8}$$

Decoupled gain matrix for the real power measurements is then formed as:

$$G_{PP}=H_{PP}^{T}\cdot H_{PP}=\begin{bmatrix}
3 & 0 & -1 & 1 & -4 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 2 & 0 & 1 & -4 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
-1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
1 & 1 & 0 & 4 & -4 & 1 & -3 & 1 & -3 & 1 & 0 & 0 & 0 & 1\\
-4 & -4 & 0 & -4 & 16 & -4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
1 & 1 & 0 & 1 & -4 & 3 & 0 & 0 & 0 & 1 & -2 & -2 & 1 & 0\\
0 & 0 & 0 & -3 & 0 & 0 & 14 & -5 & -8 & 1 & 0 & 0 & 0 & 1\\
0 & 0 & 0 & 1 & 0 & 0 & -5 & 3 & 1 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & -3 & 0 & 0 & -8 & 1 & 20 & -6 & 1 & 0 & 0 & -5\\
0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & -6 & 6 & -4 & 0 & 0 & 1\\
0 & 0 & 0 & 0 & 0 & -2 & 0 & 0 & 1 & -4 & 5 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & -2 & 0 & 0 & 0 & 0 & 0 & 4 & -2 & 0\\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & -2 & 1 & 0\\
0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & -5 & 1 & 0 & 0 & 0 & 2
\end{bmatrix} \tag{9}$$

Since the phase angle of the slack bus is included the matrix is singular, thus it is factorized and the zero pivots are replaced by 1, resulting in (10).

$$L=\begin{bmatrix}
1.7321 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 1.4142 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
-0.57735 & 0 & 0.8165 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0.57735 & 0.70711 & 0.40825 & 1.7321 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
-2.3094 & -2.8284 & -1.633 & 1.28E-16 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0.57735 & 0.70711 & 0.40825 & -3.20E-17 & 0 & 1.4142 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & -1.7321 & 0 & -3.93E-17 & 3.3166 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0.57735 & 0 & 1.31E-17 & -1.206 & 1.101 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & -1.7321 & 0 & -3.93E-17 & -3.3166 & -1.8166 & 1.6432 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0.57735 & 0 & 0.70711 & 0.60302 & 0.35781 & -1.4302 & 1.6216 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & -1.4142 & -1.67E-17 & -1.53E-18 & 0.60858 & -1.3133 & 0.95128 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & -1.4142 & -1.67E-17 & -1.53E-18 & -6.93E-17 & 0.61667 & -1.2511 & 0.2334 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0.70711 & 8.37E-18 & 7.64E-19 & 3.46E-17 & -0.30833 & 0.62555 & -0.1167 & 1 & 0\\
0 & 0 & 0 & 0.57735 & 0 & 1.31E-17 & 0.60302 & 0.35781 & -0.82158 & -0.61667 & -0.32573 & -0.1167 & 0 & 1
\end{bmatrix} \tag{10}$$

The corresponding entry of the right hand side vector t_P is assigned arbitrary values in the position of zero pivots:

assigned to t_P in the position of zero pivots

$$t_P = \begin{bmatrix} 0 & 0 & 0 & 0 & \boxed{0} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \boxed{1} & \boxed{2} \end{bmatrix}^T \tag{11}$$

$$G_{PP}^{-1} \cdot t_P = \left(L \cdot L^T\right)^{-1} \cdot t_P = \begin{bmatrix} -2 & -2 & -2 & 2 & 0 & 2 & 2 & 2 & 2 & 2 & 2 & 1.5 & 1 & 2 \end{bmatrix}^T \tag{12}$$

The estimated flows on the branches are obtained through:

$$P_b = \begin{bmatrix} 0 & -2 & 0 & -4 & -2 & -4 & 2 & 0 & 0 & -2 & 0 & 0.5 & 1 & 0 & 0 & 0 & 0 & 0 & 0.5 & -1 \end{bmatrix}^T \tag{13}$$

Thus the branches corresponding to position 2, 4, 5, 6, 7, 10, 12, 13, 19 and 20 of vector P_b have non zero flows corresponding to unobservable branches. These are 1-5, 2-4, 2-5, 3-4, 4-5, 5-6, 6 12, 6 13, 12-13 e 13-14. In Fig. 2 it can be seen the observable regions of IEEE 14 that are separated by the unobservable branches identified earlier.

A similar application of the observability analysis is used on the other networks tested. The measurement configuration used on the IEEE bus test systems are synthesized in Table I. Table I also illustrates the zero injection buses of the bus test systems used. It is shown the measurement redundancy (η_{initial}) achieved with the initial configuration measurement.

TABLE I
CONFIGURATION MEASUREMENT OF BUS TEST SYSTEMS

Data				
IEEE bus test system	Network configuration	Zero injection bus	Initial configuration measurement	η_{initial}
14	14 bus 20 lines	7	Power injections: 5, 7, 8, 9, 10, 11, 12. Power flows: 1-2, 2-3, 4-7, 7-8, 7-9, 9-14.	≅1
30	30 bus 41 lines	6, 9, 22, 25,27, 28	Power injections: 1, 2, 3, 8, 9, 10, 13, 15, 24, 26, 27. Power flows: 1-2, 1-3, 2-5, 2-6, 6-28, 9-11, 10-21, 12-13, 12-16, 14-15, 15-23, 16-17, 15-18, 18-19, 22-24, 25-26, 25-27.	≅1
57	57 bus 78 lines	4, 7, 11, 21,22, 24, 26, 34, 36, 37,39, 40, 45, 46, 48	Power injections: 1, 3, 5, 7, 8, 10, 11, 12, 13, 16, 17, 20, 21, 29, 30, 31, 33, 34, 37, 38, 39, 44, 46, 48, 49, 52, 54, 55, 56, 57. Power flows: 2-3, 3-15, 4-5,6-7, 6-8, 9-11, 9-55, 11-13, 14-15, 15-45, 18-19, 20-21, 22-38, 23-24, 25-30, 26-27, 27-28, 32-33, 35-36, 37-38, 40-56, 41-42, 41-43, 50-51.	≅1

Table II resumes the results obtained from the performance of an observability analysis.

TABLE II
OBSERVABILITY ANALISYS RESULTS

IEEE bus test system	Unobservable branches	Results
14	1-5, 2-4, 2-5, 3-4, 4-5, 5-6, 6 12, 6 13, 12-13 e 13-14	Fig.2
30	4-12, 5-7, 6-7, 6-9, 6-10, 9-10, 10-17, 10-20, 10-22, 19-20, 21-22, 23-24, 24-25, 27-28, 27-30, 29-30	Fig.3
57	1-2, 1-15, 1-16, 1-17, 4-18, 9-10, 9-12, 10-12, 10-51, 12 13, 12-16, 12-17, 13-49, 14-46, 22-23, 24-25, 24-26, 32-34, 34-35, 36-37, 36-40, 37 39, 38 44, 38 48, 38-49, 39-57, 41-56, 42-56, 44-45, 46-47, 47-48, 48-49, 49-50, 56-57	Fig.4

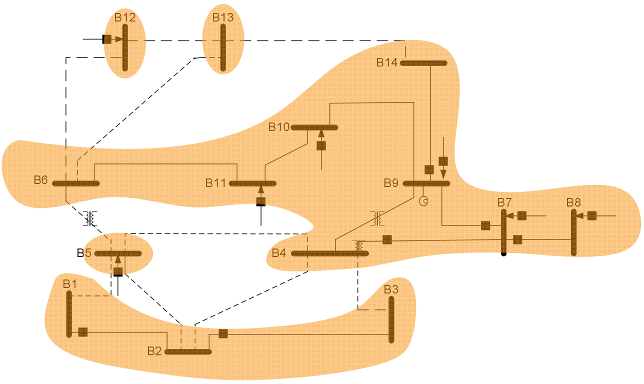

Fig. 2. Observable regions of IEEE 14 bus system example

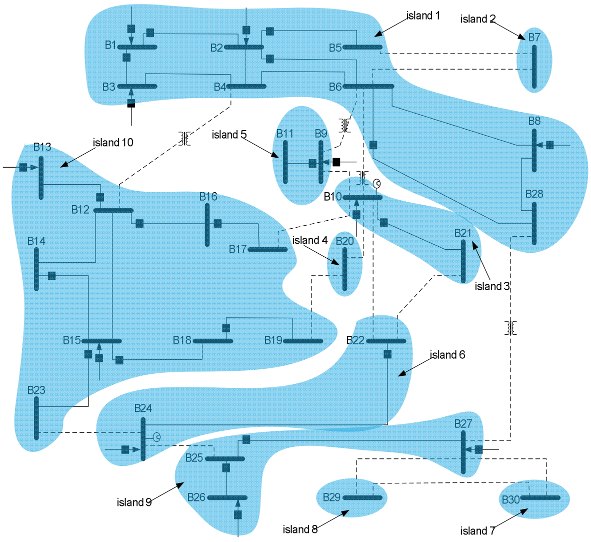

Fig. 3. Observable regions of IEEE 30 bus system example

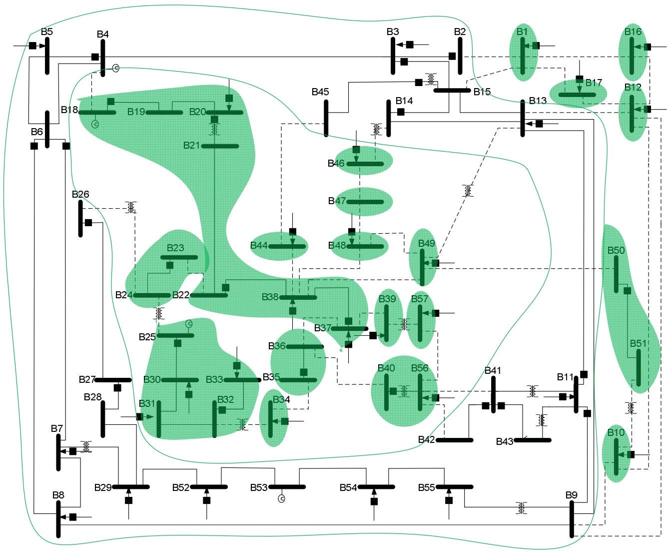

Fig. 4. Observable regions of IEEE 57 bus system example

Once identified the network observable islands, other measures may be added to the measurement set in order to join these observable regions and establish a single observable region. These measures correspond to flows in the identified unobservable branches and bus injections at buses that are on the border of the observable islands.

It was used measures from available PMUs located at the selected buses .Table III shows, for each bus test system, the PMU used, the measurements added to the conventional measurement set, the new measurement redundancy (η_{new}) and the observability results.

The use of PMU at zero injection buses for network observability has the effect of being necessary the use of less PMUs to reestablish system observability. As the intent is to minimize the number of PMUs placed in an N bus system, the use of PMU located at zero injection buses, buses with large number of connected branches, will maximize the coverage allowing the use of a minimal PMU set.

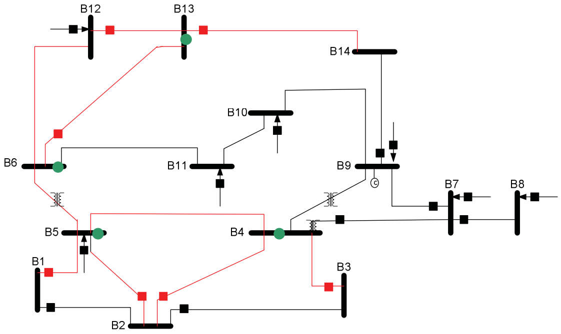

Fig. 5. Full observable IEEE 14 bus system example

The measurement redundancy clearly increases with the use of available measures from PMUs. The results obtained on the various test systems have shown that about 30% of the system buses need to be provided with PMU to ensure system observability reestablishment.

TABLE III
MEASUREMENTS USED FOR SYSTEM OBSERVABILITY

N bus	PMU	% of buses with PMU	Nº of zero injection buses with PMU	Data		
				Measures added	η_{new}	Obser-vability results
14	4, 5, 6, 13	28.5%	0	Current flow: 1-5, 2-4, 2-5, 3-4, 5-6, 6-13, 12-13, 13-14	1.7	Fig.5
30	4, 5, 6, 10, 19, 21, 24, 27, 29	30%	2	Current injection: 4, 5, 19, 21, 24, 29 Current flow: 4-12, 5-7, 6-7, 10-17, 10-20, 10-22, 19-20, 21-22, 23-24, 24-25, 27-28, 27-29, 27-30, 29-30	1.6	Fig.6
57	1,4, 10, 12, 14, 22, 24, 34, 36, 38, 39, 47, 49, 56	26.3%	6	Current injection:1, 10, 12, 14, 38, 47, 49, 56 Current flow: 1-2, 1-15, 1-16, 1-17, 4-18, 9-10, 9-12, 10-12, 10-51, 12-13, 12-16, 12-17, 13-49, 14-46, 22-23, 24-25, 24-26, 32-34, 34-35, 36-37, 36-40, 37-39, 38-44, 38-48, 38-49, 39-57, 41-56, 42-56, 44-45, 46-47, 47-48, 48-49, 49-50, 56-57	1.6	Fig.7

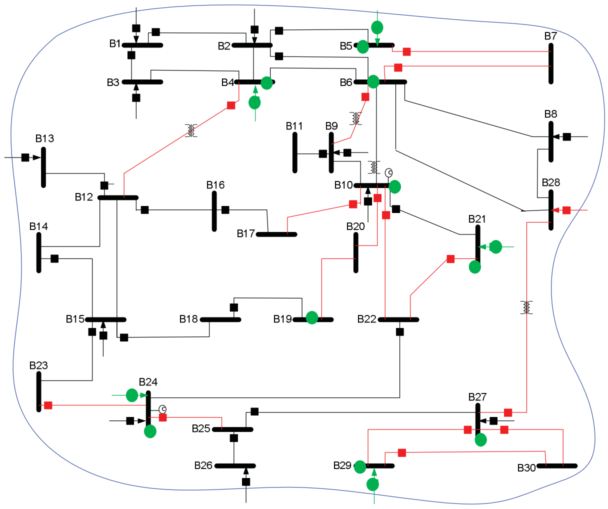

Fig. 6. Full observable IEEE 30 bus system example

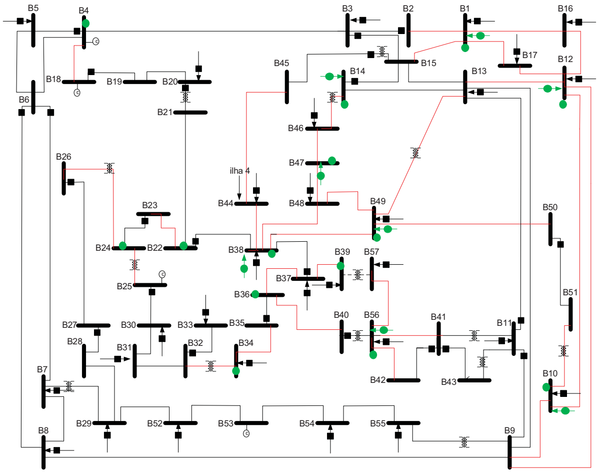

Fig. 7. Full observable IEEE 57 bus system example

IV. CONCLUSION

An approach for analyzing and making use of PMUs measurements to ensure full system observability is presented. It makes used of available PMUs measurements to define the measurements that are candidate to reestablish system observability. Although this approach does not yet returns an optimal solution that minimizes the number of PMUs placed, nevertheless it can be use to obtain system observability as the power systems are becoming more populated by PMUs.

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