MATHEMATICAL MAGIC IN THE CLASSROOM

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Abstract

Mathematics, considered one of the basic areas of various formations, has been the subject of concern for many authors and researchers due to its enormous academic and educational failure [1]. Its importance in day-to-day life and the formation of individuals is irrefutable [2]. One of the problems identified in the failure of mathematics teaching is the lack of motivation that students feel towards the discipline. According to [3], motivation is an essential factor in any learning since the quality of learning is not only related to the capacity to learn, but also to the level of motivation that we have to carry out this same learning. Considering that it is imperative to make the teaching and learning process of mathematics more stimulating, taking into account modern society and student's interests [4], the authors have developed a qualitative case study to evaluate how "mathematical magic" can contribute to increase students' motivation for learning mathematics. In order to develop this experience, the techniques of inquiry, direct observation and analysis of documents were applied and the following instruments were used: questionnaires and respective analysis grids; production of a battery of tasks of a diversified nature; field notes and interviews.

Although this project is still ongoing and therefore not yet completed, a previous analysis of the collected data allows us to conclude that the use of mathematical magic tricks in the classroom, with the purpose of motivating the students to learn mathematics, was revealed effective. Students were curious about the new tricks and were positively surprised at the relative simplicity of their explanation, as if saying to themselves, "How can such mathematical concepts bring so much surprise?" The students showed that the topics gained more meaning after performing the tricks because they witnessed real applications of the concepts, with an extremely attractive purpose. They often stated that they were going to play the tricks on their friends / family outside the classroom context, which meant that they were mastering the concepts involved.

Keywords: mathematics, mathematics education, recreational mathematics, mathematical magic, higher education.

1 INTRODUCTION

Mathematics, considered one of the basic areas of various formations, has been the subject of concern for many authors and researchers due to its enormous academic and educational failure [1]. Its importance in day-to-day life and the education of individuals is irrefutable [2]. One of the problems identified in the failure of mathematics teaching is the lack of motivation that students feel towards the subject. According to [3], motivation is an essential factor in any learning since the quality of learning is not only related to the capacity to learn, but also to the level of motivation that we have to carry out this same learning.

The Joint Mathematical Council of the United Kingdom [4] states that it is of the utmost importance that the teaching and learning process of mathematics becomes more stimulating, taking into account modern society and the students' interests.

The present world is constantly changing. New challenges emerge every day. Students should be prepared to face the uncertainty and constant change of the future. However, the density and rigidity of most math curricula are directed in the opposite direction of what is needed. Nassim Taleb [5] points out the importance of the unexpected in today's world. The author argues that human beings tend to focus on what they already know, preferring the particular over the general. As in many other contexts, school curricula reflect our tendency to control and predict. Elliot Eisner [6] argues that, although understandable at a practical level, this tendency must be inverted: "Opening oneself to the uncertain is not a pervasive quality of our current educational environment. I believe it needs to be among the values we cherish. … How can the pursuit of surprise be promoted in a classroom?" (p.11).

Being a teacher in the current context poses challenges that go beyond a solid knowledge of the subjects to be taught and the didactic issues associated with them. Being a teacher also means being creative in finding appropriate answers for today's students, who, due to being exposed to an immense diversity of sources of information and ways of occupying their time, impose an increased demand on the educational actors, making the teaching / learning process more exigent than ever before.

Being mathematics teachers, both authors of this paper are confronted, year after year, with students' lack of motivation, detachment and failure in relation to mathematics subjects. Thus, they decided to use mathematical magic tricks in their classes to arouse the interest of the students, captivating them and stimulating the like for mathematics. In this sense, they have developed a qualitative case study to evaluate how mathematical magic can contribute to increase students' motivation regarding mathematics learning.

2 MATHEMATICAL MAGIC AND MATHEMATICS EDUCATION

The issue of academic failure in mathematics is at the order of the day. Knowing its causes and finding ways to combat it is a priority at all levels of education. Year after year, teachers are faced with discouragement, detachment and consequent student failure. It is imperative to change the educational model ([7], [8], [9], [10], [11], [12], [13], among others); It is necessary to have another look at the way in which one learns and, consequently, how one must teach ([7], [14], [11], [12], [13], [14]). Researchers, such as Nóvoa [15], argue that the educational process should be focused on the student's learning. The student is challenged to abandon his role as a passive recipient of information and to assume a central role in the teaching/learning process, adopting a more critical, active, regulatory and dynamic attitude in the personal construction of his own knowledge. Reversing the negative general feeling toward mathematics is essential. It is crucial to motivate, arouse interest, thus promoting students' success in mathematics. One way to do this is through recreational mathematics, which David Singmaster [16] defines as math that is fun, popular, and of pedagogical value. There are several areas of mathematics in which recreation and seriousness are easily interlinked (as for instance through games): probability, graph theory, number theory, topology, geometry, among others.

Recreational mathematics is quite often linked to game theory. Mathematical magic is another of its important facets. At first glance, mathematics and magic do not seem to have much in common, but, looking more closely, we realize that this relationship has been around for a long time with valuable benefits for both parties. Five hundred years ago, Luca Pacioli described the first math-related card trick in his manuscript De Viribus Quantitatis [17]. Since then, much work has been developed applying mathematical ideas in creating different types of magic tricks, including numerical tricks, card tricks, and visual effects based on geometric relationships or topological features. Among the authors of the magic world, Martin Gardner (1914-2010) deserves special mention for his important role in building bridges between the world of magic and mathematics. Martin Gardner published close to 200 books, including the book Math, Magic and Mystery [18], first published in 1956 by Dover, which has influenced so much magicians and mathematicians ever since. Colm Mulcahy is one of the mathematicians inspired by Martin Gardner. Mulcahy created a bimonthly column, Card Colm, on the American Mathematical Association's website, from 2004 to 2014, exclusively dedicated to mathematical card tricks (http://cardcolm.org/CardColms.html). More recently, he published a book where he gathered many of the effects described in his column [19].

Several other mathematicians have dedicated themselves to the dissemination of mathematical magic and its promotion both to its students and to society in general. Diaconis and Graham [20] in Canada have published a wonderful book that stands out for the mathematical depth of their magic tricks. Marcus du Sautoy of Oxford University has a group in the UK, Marcus' Marvellous Mathemagicians, who visit schools and science museums communicating mathematics through magic (https://www.maths.ox.ac.uk/study-here / undergraduate-study / outreach / marcus-marvelous-mathemagicians). Fernando Blasco of the Polytechnic University of Madrid holds numerous sessions of mathematical magic. Blasco has a website and a blog to share his ideas (http://www.fblasco.com/index.html) and published a book on mathematical magic [21]. On his website, Blasco says: "I am fully convinced that if everyone knew more details about what mathematics involves, they would become interested in this subject. ... The world of mathematics offers as much seriousness and rigor as illusion and entertainment. Many are convinced that mathematics is something austere and boring, but in spite of these strong convictions, things are not always what they seem. "Magic is just that: to make things look like what they are not, or to make look

differently from what they are, producing constant surprise, wonder and fun. Ultimately, the astonishment is that all of this can be done using mathematics.

Exploring the relationship between magic and mathematics has an enormous potential in developing activities in the classroom to promote the interest in mathematics. Performing a magic trick and exploration its mathematical content can be a source of reasoning and mathematical learning which can be empowered by the magic setup.

3 A SPECIFIC CASE

3.1 Description of the study

A research project is in course with the main goal of evaluating the impact of the use of mathematical magic in the classroom in the learning of Mathematics by pre-service teachers. The underlying research question is whether mathematical magic can contribute to increase students' motivation regarding mathematics learning.

The empirical part, admitted as methodological options a case study in an action-research context.

The study is an ongoing project involving pre-service teachers who are doing their undergraduate degree – Licenciatura em Educação Básica (LEB) (*Bachelor's degree on Basic Education*) - at a Portuguese university. One of the researchers of this study lectures one of the mathematics courses of this degree – Conceitos de Matemática I (CM I) (*Mathematical Concepts I*) and this was the setup chosen for the study. This course covers several topics related to the general theme of numbers and operations present in the elementary and middle school mathematics syllabus (the decimal system and the representation of numbers in base 10 and in other bases; operations with integers; an introduction to number theory; operations with rational numbers).

As a starting point, previous to the beginning of the present term (2017-18), the course was planned, as well as the mathematical magic tricks to be use in the classroom. Whenever possible the mathematical content of the tricks was related to the topics taught in the class (eg. changing bases, operations algorithms, and divisibility rules). At the beginning of the term, a first questionnaire was delivered to the students. The main objective of this questionnaire was to characterize the students, because it was fundamental to know their background.

The next step was the didactic approach in class. Every class, about 10 minutes were reserved (at the beginning or at the end) to perform a trick. In general, the explanation of the trick was only discussed in the following class in order to give the students the opportunity to explore the mathematical content by themselves.

During this phase, the data collecting was made through direct observation and documental analysis, supported by the following instruments: field notes and students portfolio.

Towards the end of the term, a second questionnaire was applied. This time the goal was to collect the students' opinion on the use of mathematical magic in the classroom.

In the next stage, interviews will also be held, to clarify, with the case-students, any doubts that the researchers may still have.

3.2 Some results

In this section we report a preliminary analysis of the questionnaires mentioned previously as well as some of the results obtained through the field notes and students' portfolios.

Regarding the first questionnaire of characterization the results show that most students are female (93% out of 70 respondents). Most students (75%) chose languages and humanities are their main topics in secondary school, only 11.4% chose science and technology, 4.3% socioeconomics and 7.1% chose professional courses.

Only 23% of the respondents had mathematics on the last year of secondary school (12^{th} grade) and only 36% had mathematics during the 10^{th} and 11^{th} grades. It should be noted that 37.1% of the respondents did not have any mathematics course in secondary school.

When asked if they considered themselves to be good students in mathematics, only 1 student of the 70 respondents said that he considered himself to be very good. Most students (50%) considered

themselves to be reasonable, with the remaining 48.5% considering themselves to be weak or very weak (31.4% weak and 17.1% very weak).

This characterization does not differ greatly from the past and these facts are consistent with the fact that most pre-service teachers reveal many difficulties in mathematics and are not motivated for the subject. This is one of the main reasons why the authors have undertaken this project, in order to improve the students' attitude towards mathematics and promote a more efficient learning process.

Regarding the second questionnaire, delivered towards the end of the term, students were asked (in a five point Likert scale: 1- not at all, 5- very much) if they enjoyed the performance of mathematical magic trick in the classes. The majority of the students gave a positive reply: 70,4% replied 4 - "I liked" and 25,9% replied 5 - "very much". Only one student (out of 53 who filled in the questionnaire) replied 3 - "indifferent", and none gave a negative answer (2- "not much" or 1 – not at all").

Students were also asked to rate in a 5 point Likert scale their agreement regarding several statements. One of the statements was "The use of mathematical magic in the classroom contributes to a more positive attitude towards mathematics". About 85% of the students gave a positive answer, as can be seen in Table 1. Another of these statements was "The use of mathematical magic tricks in math classes makes mathematics more boring and demotivating". None of the students agreed with the statement as can been seen in Table 1.

Table 1. *Distribution (in %) of the answers on the agreement with some given statements (1 – total disagreement; 3 – neutral; 5 – total agreement)*

Statement	1	2	3	4	5
The use of mathematical magic tricks in math classes contributes to a more positive attitude towards mathematics.	3.7%	1.8%	9.3%	42.6%	42.6%
The use of mathematical magic tricks in math classes makes mathematics more boring and demotivating.	82.7%	13.4%	3.9%	0%	0%

When questioned about whether the use of magic tricks in mathematics classes is considered important, 56.4% of the respondents said to be "important", 28.2% said "very important" and only 15.4% said "indifferent". No student believes it is "not important" or "not at all important".

When asked if the use of magic tricks in math classes helped them to enjoy mathematics more, the great majority of the students - 69% - answered "Yes" (31% answered "No").

A preliminary analysis of the field notes and the students' portfolios allows us to strike out the following:

- Over the course of the lessons the researcher/teacher could observe that the students were curious about new tricks and were positively surprised by the relative simplicity of their mathematical explanation, as if to say to themselves "how can mathematical concepts surprise us so much? ".
- All the tricks involved volunteers. In all classes there were always several students very interested in volunteering, which reveals that the students liked the activity and liked to participate in it.
- Quite often several students would think about the trick, after its performance, trying to figure out what was behind it. They explicitly asked the teacher not to reveal the secret immediately so they could think on it. At other times the curiosity was so strong that they asked the teacher to explain it immediately. Either way the students were curious and were developing their mathematical reasoning and engaging with mathematics.
- The students showed that the subjects gained more meaning after performing the tricks because they had concrete examples of the application of the concepts, with an extremely attractive purpose.
- Several times students said that they would play the tricks with their friends / family, outside the classroom context, showing that they were motivated and that they were mastering the application of the concepts involved.
- Whenever the teacher forgot or intentionally left the trick for the end of the lesson, the students inquired if the teacher had forgotten the trick for this class.

4 CONCLUDING REMARKS

Finding ways to engage students in the classroom and motivate them for learning mathematics is a challenge that most mathematics teachers face today. All contributions towards this goal are welcome and should be disseminated across these professional. There is no single solution to this challenge and as in many other complex problems, several solutions must be combined and adapted to each context. In this paper the authors propose one particular way to contribute to this goal: using mathematical magic tricks in the classroom.

Although this project is still ongoing, therefore, not yet been concluded, some conclusion can already me drawn. The analysis of the collected information, through the questionnaire applied in the end of the term, the field notes and the students' portfolios, allows us to conclude that the use of mathematical magic tricks in the classroom, with the aim of motivating students to learn mathematics, has proved effective in math courses for pre-service teachers, namely CM I within the undergraduate degree LEB in a Portuguese university.

This project leads us to believe that the use of tricks with mathematical magic on a regular basis throughout most math courses is an added value in the process of learning with positive effects on both the motivation for learning and the learning itself.

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