RESEARCH ARTICLE | DECEMBER 31 2015

Searching for the corner seismic moment in worldwide data

Miguel Felgueiras; Rui Santos; João Paulo Martins

AIP Conf. Proc. 1702, 030003 (2015)
https://doi.org/10.1063/1.4938769

Searching for the Corner Seismic Moment in Worldwide Data

Miguel Felgueiras, Rui Santos and João Paulo Martins

CEAUL Lisbon and ESTG, Polytechnic Institute of Leiria, Portugal

Abstract. In this paper the existence of the corner frequency value for the seismic moment distribution is investigated, analysing worldwide data. Pareto based distributions, usually considered as the most suitable to this type of data, are fitted to the most recent data, available in a global earthquake catalog. Despite the undeniable finite nature of the seismic moment data, we conclude that no corner frequency can be established considering the available data set.

Keywords: Pareto based distributions, seismic moment, corner frequency.
AMS: 60E05; 62P35.

INTRODUCTION

Great earthquakes are a rare phenomenon. The search of physical laws to explain the energy released by them (seismic moment) is an important issue, since they usually cause heavy losses. Several works have been produced on this matter, as [6, 11] concerning to the physical phenomenon point of view, or [4, 8] concerning to the statistical point of view.

Usually, the statistical models for the seismic moment are found in the heavy tailed class, specially in the Pareto based distribution family. In fact, there is a physical reason for this choice, since the seismic moment can be converted into a power law [12, 19]. Previous works [9, 14, 20] point to a Pareto distribution with shape parameter $\alpha \approx 0.6667$.

The demand for a corner frequency value for the seismic moment data, that is, a magnitude value that splits large from great earthquakes, is a particular problem in this subject. It is also connected with the maximum magnitude search. Clearly, the energy released in a seism must be finite, however the behaviour of the great earthquakes is still unclear (basically because they are rare and complex to measure in an exact way [11]). Great earthquakes are important since they can cause heavy losses. As far as we know, a right end-point in the support of the magnitude distribution (the size of tectonics plates is finite) is still unknown, but should probably be around $M_W \approx 10$ [10, 1] for world data, where M_W is the Moment Magnitude scale.

Usually, modelling the energy released in the higher magnitude earthquakes shows lack of fit whenever catalogs for some particular seismic regions are considered. An exponential tapper (the existence of characteristic scales, like the thickness of the seismogenic crust, may imply non self similarity between small and large events) is applied to the right tail of the distribution by some authors, while others consider two power laws with different shape parameters to explain the entire data set [11, 16]. However, [4, 20] note that some seismic regions may have the reverse situation, that is, the Pareto distribution possibly underestimates the frequency of the very large earthquakes. [15] establishes $M_W = 8.1 \pm 0.3$ as the corner frequency value for the global earthquake catalog, splitting large from great earthquakes. Since new and more accurate data is now available (see page 3), it is pertinent to restudy this matter, in order to verify the estimative of the corner frequency value or even to validate its existence.

SEISMIC MOMENT AND MOMENT MAGNITUDE SCALE

Nowadays, the M_W is the more often used scale to measure the intensity of an earthquake, since it does not saturate for large events. Unfortunately, only events observed after the beginning of the last century have a reliable estimate of this magnitude. The energy released by an earthquake, measured by its seismic moment M_0, depends on the area of fault rupture, the average value of the final slip, the rigidity modulus of the rocks and other material surrounding the fault [11]. Nevertheless, since the previous data is not often available, we can connect M_W and M_0 by [6]

$$M_W = \frac{\log_{10} M_0 - 9.1}{1.5} \iff M_0 = 10^{1.5M_W+9.1}$$

International Conference of Computational Methods in Sciences and Engineering 2015 (ICCMSE 2015)
AIP Conf. Proc. 1702, 030003-1–030003-4; doi: 10.1063/1.4938769

assuming that M_0 is in Newton-meter (Nm). Indeed, there is a great difference in the energy released by an earthquake even when the increase in M_W is small. If M_W increases 0.1 then M_0 increases 41.25%. Thus, an earthquake with magnitude $M_W = 9.5$ releases the same energy than 178 earthquakes with magnitude $M_W = 8.0$.

HEAVY AND SUPER-HEAVY TAILED MODELS FOR THE SEISMIC MOMENT DATA

In this section we introduce some heavy or super-heavy tailed models that seems to be adequate to model the seismic moment. Pareto distribution is the most widely used distribution in this context, as stated before, and estimation issues and characterizations can be found in [7]. [13] studied and applied the super heavy tailed log-Pareto, and in [4] and [5] were introduced and analysed others Pareto based distributions which have proved to be accurate to fit this kind of data. In fact, since we are only interested in the great seisms, a truncation point is necessary to exclude the smaller events.

Pareto Distribution

The truncated Pareto distribution considered in this work only has a shape parameter $\alpha > 0$ and a truncation point $k > 1$. Therefore, it has distribution function

$$F_{Y|Y>k}(x) = 1 - \left(\frac{x}{k}\right)^{-\alpha}, \quad x \geq k.$$

A key point in the Pareto distribution is its self-similarity, implying that

$$\frac{Y|_{Y>k}}{k} \sim Pareto(\alpha).$$

Besides, Pareto distribution is scale free, meaning that whatever scale we look at in, the scale parameter is the same.

Log-Pareto Distribution

For the truncated log-Pareto with shape parameter $\alpha > 0$ and truncation point $k > 1$, the distribution function is

$$F_{Y|Y>k}(x) = 1 - \left(\frac{\ln x}{\ln k}\right)^{-\alpha}, \quad x \geq k.$$

Both Pareto and log-Pareto truncated distributions can be transformed into an exponential distribution, allowing us to benefit from the characterizations of this family.

Location Scale Pareto Mixture Distribution

A location scale Pareto mixture (LSPM) distribution can be defined as

$$W = \mu + \sigma Y = \mu + \sigma \Theta X, \quad \mu \in \Re, \sigma > 0$$

where μ and σ are location and scale parameters, $\Theta \sim Pareto(\alpha)$ and $X \sim Pareto(1)$ are independent random variables. The support of the random variable W is $S_{\mu+\sigma\Theta X} = [\mu + \sigma, \infty[$ and therefore μ and σ define a truncation point. The distribution function can be obtained in a closed form as

$$F_W(x) = \frac{\alpha\left(\frac{x-\mu}{\sigma}\right)^{-2-\alpha}\left(x - \mu - \sigma\left(\frac{x-\mu}{\sigma}\right)^{\alpha}\right)}{\sigma(1-\alpha)}.$$

The Extended Slash Pareto Distribution

The extended slash family is obtained considering the independent random variable quotient

$$Y = \frac{X}{\Theta}$$

where $\Theta \sim Beta(\alpha, 1)$, with $\alpha > 0$. When $X \sim Pareto(\alpha)$ we obtain the extended slash Pareto (ESP) distribution. Considering a truncation point k, ESP distribution function is

$$F_{Y|Y>k}(x) = 1 - \frac{1+\alpha \ln x}{1+\alpha \ln k} \left(\frac{x}{k}\right)^{-\alpha}, \quad x \geq k \geq 1.$$

THE ANALYSED DATA SET

The International Seismological Centre (ISC), under request of the Global Earthquake Model (GEM), produced and released the ISC-GEM Global Instrumental Earthquake Catalogue in 02-2014 (`http://www.globalquakemodel.org/`). This recent dataset is considered more accurate than previous ones and has been used in recent works [2, 18]. It contains a total of $n = 18867$ events registered between 1900 and 2009.

As the estimated corner frequency value for the global earthquake catalog is $M_W = 8.1 \pm 0.3$ (see Introduction), we considered as truncation points $M_0 = 10^{20.8+k}$ with $k = 0, 0.1, 0.2, ..., 0.9$, corresponding to M_W between 7.8 and 8.4. It would be irrelevant to consider smaller increments of k since typically M_W is registered with one precision digit leading to uncertain in the M_0 estimation. This leads to samples with dimension between $n = 14$ and $n = 125$.

OBTAINED RESULTS

The introduced models were applied to the datasets, leading to the results displayed in Table 1, where each column represents the results for a particular M_W value. For the *LSPM*, estimation was performed using a variation from the method of moments [5]. For all the other applied models, maximum likelihood estimates were used. After that, some of the most commonly applied goodness of fit tests were used to assess the models fit, such as Anderson-Darling (AD), Cramér-von Mises (CM) and Kolmogorov-Smirnov (KS) [17]. Common model selection criterion like the Akaike Information Criterion (AIC) or the Bayesian Information Criterion (BIC), cannot be used for comparing the adjusted models since those depend on different data set transformations, and therefore are incomparable with the usual information criterion [3]. Thus, parsimonious issues are not numerically tackled in this work and therefore will depend on each scientist opinion. All the models seem well fitted to the data, except for $M_W = 8.07$ where KS test *p-value* is significant (lower than the significant level of 0.05), excluding *LSPM* model. Note that the *LSPM* distribution has three parameters instead of one like the other analysed distributions, thus is always the best fitted to the data considering KS test. In that model, the other parameters estimation led to a large variability on the α parameter estimation. More important, no break points were detected in the adjusted models, and therefore it appears to be pointless to consider a corner frequency value to split large from great earthquakes. For all models, the M_W value is irrelevant (all the $p-values > 0.4$) in a regression model predicting α as a linear function of M_W, despite a slightly negative trend. It is also noteworthy to mention that this new catalog led to estimatives of α (for the Pareto model) larger than the previous ones, obtained with older catalogs (see Introduction) implying lighter tail weight.

CONCLUSION

Although the seismic moment of a seism must be finite, it is yet to be found a reasonable estimate to this value. Moreover, the obtained results with a recent and more reliable data set show that there is no evidence to support the idea that the behaviour of large and great earthquakes seismic moment is different, and therefore the analyzed data do not allow to find out a corner frequency value statistically significant. Hence, there is no need to consider different models (or at least different parameters of the same models) to model this kind of data, due to lack of evidence to sustain the splitting between large and great earthquakes.

TABLE 1. Estimated shape parameters (in bold) and goodness of fit *p-values*. For each column, the combination M_W/ model that better fit the data (higher *p-value*) is bold written.

M_w	7.80	7.87	7.93	8.00	8.07	8.13	8.20	8.27	8.33	8.40
M_0	$10^{20.8}$	$10^{20.9}$	$10^{21.0}$	$10^{21.1}$	$10^{21.2}$	$10^{21.3}$	$10^{21.4}$	$10^{21.5}$	$10^{21.6}$	$10^{21.7}$
n	125	110	86	70	65	40	31	28	18	14
Pareto α estimative	**0.910**	**0.997**	**0.975**	**1.009**	**1.204**	**0.950**	**0.935**	**1.068**	**0.861**	**0.826**
AD *p-value*	0.559	**0.882**	**0.499**	**0.459**	0.255	0.765	**0.788**	0.618	0.978	0.742
CM *p-value*	0.379	0.503	**0.309**	**0.245**	0.130	**0.588**	**0.572**	0.405	**0.837**	0.654
KS *p-value*	0.479	**0.499**	0.338	0.224	0.034	0.720	0.773	0.518	0.959	0.886
Log-Pareto α estimative	**44.472**	**48.950**	**48.100**	**50.025**	**59.993**	**47.547**	**47.053**	**54.002**	**43.790**	**42.161**
AD *p-value*	0.491	0.852	0.483	0.443	0.265	0.726	**0.788**	0.618	0.959	**0.787**
CM *p-value*	0.334	0.472	0.298	0.240	0.134	0.573	**0.572**	0.405	0.818	**0.712**
KS *p-value*	0.477	0.397	0.251	0.224	0.034	0.727	0.773	0.518	0.959	0.886
LSPM α estimative	**1.255**	**3.351**	**1.038**	**1.079**	**2.732**	**1.259**	**1.176**	**2.472**	**2.003**	**1.100**
AD *p-value*	0.455	0.879	0.256	0.289	**0.479**	**0.782**	0.714	**0.677**	**0.979**	0.650
CM *p-value*	0.369	**0.504**	0.200	0.178	**0.206**	0.549	0.481	**0.439**	**0.837**	0.551
KS *p-value*	**0.580**	**0.499**	**0.442**	**0.225**	**0.087**	**0.888**	**0.778**	**0.523**	**0.965**	**0.902**
ESP α estimative	**0.930**	**1.017**	**0.995**	**1.029**	**1.224**	**0.969**	**0.954**	**1.087**	**0.881**	**0.845**
AD *p-value*	**0.576**	**0.882**	**0.499**	**0.459**	0.255	0.765	**0.788**	0.618	0.978	0.742
CM *p-value*	**0.388**	0.503	**0.309**	**0.245**	0.130	**0.588**	**0.572**	0.405	**0.837**	0.654
KS *p-value*	0.478	**0.499**	0.338	0.224	0.034	0.720	0.773	0.518	0.959	0.886

ACKNOWLEDGMENTS

Research partially funded by FCT, Portugal, through the project Pest-OE/MAT/UI0006/2014.

REFERENCES

1. P. Bird, Y. Kagan, Plate-Tectonic Analysis of Shallow Seismicity: Apparent Boundary Width, Beta, Corner Magnitude, Coupled Lithosphere Thickness, and Coupling in Seven Tectonic Settings, *Bull Seismol Soc Am* **94** 6 2380–2399 (2004).
2. A. Bocchini, E. Pomonis, E. So, A. King, S. Giovinazzi, The GEM earthquake consequences database and New Zealand's contributions, *2014 NZSEE Conference* (2014).
3. K. Burnham, D. Anderson, *Model Selection and Multimodel Inference*, Springer (2002).
4. M. Felgueiras, Explaining the seismic moment of large earthquakes by heavy and extremely heavy tailed models, *Int J Geomath* **3-2** 209-222 (2012).
5. M. Felgueiras, R. Santos, Exploring Pareto Scale Mixtures, *AIP Conf Proc* **1479** 1121-1124 (2012).
6. T. Hanks, H. Kanamori, A Moment Magnitude Scale, *J Geophys Res* **84** 2348-2350 (1979).
7. N. Johnson, S. Kotz, N. Balakrishnan, *Continuous Univariate Distributions, Volume I*, Wiley (1994).
8. Y. Kagan, Earthquake Slip Distribution: A Statistical Model, *J Geophys Res* **110** 1-15 (2005).
9. H. Kanamori, The Energy Release in Great Earthquakes, *J Geophys Res* **82** 2981-2987 (1977).
10. A. Kijko, Estimation of the Maximum Earthquake Magnitude, mmax, *Pure Appl Geophys* **161** 1655-1681 (2004).
11. Madariaga, R. Earthquake Scaling Laws. *Encyclopedia of Complexity and Systems* 2581-2599 Springer (2009).
12. Newman, M. Power laws, Pareto distributions and Zipf's law. *Contemp Phys* **46(5)** 323-351 (2005).
13. C. Neves, M.I. Fraga Alves, Ratio of Maximum to the Sum for Testing Super Heavy Tails, *Advances in Mathematical and Statistical Modeling* **181-194** Birkhauser Boston (2008).
14. E. Okal, B. Romanowicz, On the variation of b-values with earthquake size. *Phys Earth Planet In* **87** 55-76 (1994).
15. V. Pisarenko, D. Sornette, Statistical detection and Characterization of a deviation from the Gutenberg-Richter distribution above Magnitude 8, *Pure Appl Geophys* **161** 839-864 (2004).
16. V. Pisarenko, A. Sornette, D. Sornette, M. Rodkin, Characterization of the tail of the distribution of earthquake magnitudes by combining the GEV and GPD descriptions of Extreme Value Theory, *Pure Appl Geophys* **171** 1599-1624 (2014).
17. N. Razali, Y. Wah, Power comparisons of Shapiro-Wilk, Kolmogorov-Smirnov, Lilliefors and Anderson-Darling tests, *Journal of statistical modeling and Analytics* **2(1)** 21-33 (2011).
18. V. Silva, H. Crowley, C. Yepes, R. Pinho, Presentation of the Openquakeengine, an open software for seismic hazard and risk assessment, *Tenth U.S. National Conference on Earthquake Engineering Frontiers of Earthquake Engineering* (2014).
19. T. Utsu, Representation and Analysis of the Earthquake Size Distribution: A Historical Review and Some New Approaches, *Pure Appl Geophys* **155** 509-535 (1999).
20. I. Zaliapin, Y. Kagan, F. Schoenberg, Approximating the Distribution of Pareto Sums, *Pure Appl Geophys* **162** 1187-1228 (2005).