Advanced Time-Domain Techniques for Strongly Nonlinear RF Circuit Simulation

Recent Developments and Remaining Challenges

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Abstract—**Simulating modern wireless communication systems is today a hot topic. Serious difficulties arise when these systems are composed of strongly nonlinear highly heterogeneous circuits operating in multiple time scales. In this sense, this paper provides a general overview on the existing time-domain simulation techniques, as the ones encountered in commercial packages, or the ones recently published in the literature. Some new results will be introduced by presenting a comparison between the most relevant univariate and multivariate time-domain techniques, in terms of computational efficiency.**

Keywords-nonlinear circuits; simulation; time-domain analysis.

I. INTRODUCTION

The high heterogeneity and strong nonlinearity features of some current radio frequency (RF) networks brought a new range of challenges to circuit simulation. In effect, both the necessity of providing increased levels of wireless systems' functionality, with increased data rates, and the need to improve the transmitters efficiency and linearity, have led to an increasingly challenging scenario of heterogeneous broadband and strongly nonlinear wireless communication circuits, presenting a wide diversity of slowly varying and rapidly changing state variables. These circuits may also have excitation regimes with completely different formats and running on widely separated time scales [e.g., baseband information signals, amplitude modulated (AM) signals, phase modulated (PM) signals, RF carriers, digital clocks, pulse-width modulation (PWM) signals, etc.]. This way, integrated systems-on-a-chip (SoC), combining RF and baseband analog and digital circuitry on modern complementary metal-oxide semiconductor field effect transistor (CMOS FET) technologies, are gradually reshaping current and upcoming wireless transceiver architectures. Because the RF, the baseband and the digital blocks are intricately mixed, it is not possible to adopt the circuit-level/system-level co-simulation methodology, in which some of the blocks are represented with a simplified system-level description, while the remaining (usually the critical parts of the circuit) are simulated at the circuit-level, in a more or less independent way. Instead, a full circuit-level simulation technique is required.

At present, none of the available RF tools encountered in commercial packages are capable of simulating this kind of circuits in an efficient way. The main reason for this is that they do not perform any distinction between nodes or blocks within the circuit, regardless of their time constants and excitation regimes. This means that the same numerical algorithm is required to simultaneously compute the response of the digital blocks, the baseband analog blocks and the RF blocks. Obviously, this is undesirable since signals in different blocks have completely different features and evolve on widely disparate rates of change. For instance, it may be noted that, since time evolution rates of signals in different parts of a circuit may differ from three, or more, orders of magnitude, and the sampling rate of the signals is dictated by the fastest ones, the application of the same numerical scheme to all the blocks will result in high inefficiency. Another important aspect is that the strongly nonlinear regimes present in this kind of circuits (e.g., switching-mode operations) advise a time-domain description, in substitution of traditional frequency-domain solvers as HB.

To cope with the challenging heterogeneous nonlinear scenario described above, some innovative time-domain techniques have been proposed in the literature in the recent years [1]-[8]. Although very good results were obtained with such methods, which revealed to be appropriated to deal with some of these problems in an efficient way, there are still a few limitations that were not overcome, and so, deserving further research.

II. CLASSICAL TIME-DOMAIN SIMULATION TECHNIQUES

A. Mathematical Model of an Electronic Circuit

The dynamical behavior of electronic circuits can be described with systems of differential algebraic equations (DAE), involving voltages, currents, charges and fluxes. Such systems can be constructed from a circuit description, using, for example, nodal analysis, or modified nodal analysis, which involves applying the Kirchhoff current law to each node in the circuit, and applying the constitutive or branch equations to each circuit element. Under the quasi-static assumption, [9], [10], models this way generated have, in general, the form

$$p\left[y(t)\right]+\frac{dq\left[y(t)\right]}{dt}=x(t), \quad (1)$$

where $x(t)\in\mathbb{R}^n$ and $y(t)\in\mathbb{R}^n$ stand for the excitation (independent voltage and current sources) and state variable (node voltages and branch currents) vectors, respectively. $p\left[y(t)\right]$ represents memoryless linear or nonlinear elements, while $q\left[y(t)\right]$ models dynamic linear or nonlinear elements (voltage-dependent charges or current-dependent fluxes).

B. Transient Time-Step Integration (SPICE-like simulation)

The most natural way to evaluate $y(t)$ is to numerically time-step integrate (1) directly in time domain. One possible way to do that consists in converting the differential equations into difference equations, in which the time derivatives are approximated by appropriate incremental ratios. With this strategy, the nonlinear differential algebraic equations' system of (1) is converted into a purely nonlinear algebraic system. For example, if we discretize the time *t* using a uniform grid (a set of successive equally spaced time instants) defined as $t_i=t_0+i\cdot h$, and use the backward Euler rule (the popular implicit finite-differences scheme of order 1 [11]) to approximate the time derivatives of (1), we obtain

$$p(y_i)+\frac{q(y_i)-q(y_{i-1})}{h}=x(t_i), \quad (2)$$

where y_i denotes an approximation to the exact solution $y(t_i)$, and the parameter $h=t_i-t_{i-1}$ is the time-step integration size. Small step sizes can provide a good accuracy in the simulation results but may conduct to large computation times. On the contrary, large step sizes will reduce the computation time but will definitely conduct to poorer accuracy. A good compromise between accuracy and simulation time is achieved when *h* is dynamically selected according to the solution's rate of change.

The above formulation derives directly from the intuitive idea that derivatives can be approximated, and thus simply replaced, by finite-differences schemes. Although this technique can be used to compute the transient response of a generic electronic circuit described by (1), there is as alternative strategy which is more often employed to find the solution of initial value problems. Such strategy consists in using initial value solvers, such as linear multistep methods (e.g., Gear methods) [11], or Runge-Kutta (RK) methods [11] (the most popular time-step integrators). Both classes of these methods can provide a wide variety of explicit and implicit numerical schemes, with very distinct properties in terms of order (accuracy) and numerical stability. Consequently, for the same time-step length *h*, solutions obtained with these methods can be extremely more accurate than the ones obtained with the backward Euler differentiation rule described above in (2), which has order 1.

This straightforward technique of time-step integrating (1) directly in the time domain was used in the first digital computer programs of circuit analysis (initially developed in the early 1970's), and is still nowadays the most widely used numerical method for general purpose circuit simulation. Indeed it is present in all SPICE [12] or SPICE-like computer programs. However, its application to RF circuits may cause some problems resulting from the specific behavior of RF systems. To understand that, we must recall that RF signals are typically narrowband signals. This means that a data signal with a relatively low bandwidth is transmitted at a very high carrier frequency. To simulate a sufficient portion of the data signal, a large number of carrier periods must be time-step integrated, and thus a very large number of time samples is required (a large amount of memory and computational time consumption).

C. Periodic Steady-State Simulation (Shooting)

Computing the periodic steady-state response of an electronic circuit involves finding the initial condition, $y(t_0)$, for the differential system that describes the circuit's operation, such that the solution at the end of one period matches the initial condition, i.e., $y(t_0)=y(t_0+T)$, where T is the period. The time-domain method most commonly used for numerically evaluating the periodic steady-state solution of an electronic circuit is the shooting method [13]. Shooting solves boundary value problems by computing the solution to a succession of initial value problems with progressively improved guesses at an initial condition, which ultimately results in the periodic steady-state. In a circuit's steady-state simulation, shooting begins by time-step integrating the differential system of (1) along one period *T*, i.e., from $t=t_0$ until $t=t_0+T$, using some guessed initial condition $y(t_0)$ (generally determined from a previous DC analysis). Then, the computed solution at the end of the period $y(t_0+T)$ is checked, and if it does not agree with the initial condition, the initial condition is wisely adjusted. The circuit is then re-simulated with the updated initial condition, and this process is repeated until the solution after one period matches the initial condition. In the end, shooting relies on finding the solution of

$$y(t_0+T)-y(t_0)=0 \Leftrightarrow \phi\left[y(t_0),T\right]-y(t_0)=0, \quad (3)$$

where ϕ is the *state-transition function* [13]. An efficient way to iteratively solve (3) is to use the Newton's method.

III. Advanced Time-Domain Simulation Techniques

A. Time-Step Integration with Different Step Sizes

The operation regime of some electronic circuits involves signal excitations and/or system dynamics (time constants) of widely separate rates of variation. In such cases, node voltages or branch currents presenting slow (*latent*) and very fast (*active*) time evolution rates may coexist in the same problem. As described above, time-step integration is a classical technique that is still used nowadays by all SPICE-like computer programs for simulating electronic circuits. However, when integrating systems whose components (state variables) evolve according to different time scales one would like to use numerical schemes that do not expend unnecessary work on slowly changing components. In fact, in such cases traditional time-step integrators, like standard Runge-Kutta or linear multistep methods – which use the same step size for all system's components – become inefficient. To cope with this, some modern multirate Runge-Kutta (MRK) schemes [2], [3], have been proposed in the recent years. These powerful schemes split the differential system into coupled active and latent subsystems, to then integrate the fast varying components with a small step size and the slowly varying ones with a much larger step length.

B. Multivariate Formulation

Signals containing components that vary at two or more widely separated rates (e.g., a very fast rectangular wave with a slowly changing duty cycle, a high frequency sinusoid with a slowly varying envelope, etc.) arise in many kind of electronic circuits, such as switched-power converters, switched-capacitor filters, voltage-controlled oscillators, pulse-width modulators, phase locked loops, etc., and have a special incidence in RF and microwave communication applications, such as, modulators or demodulators, mixers (up/down converters), switched-mode power amplifiers, etc. Circuits like these are very difficult to simulate using standard simulation techniques. An efficient strategy to deal with this kind of problems consists in using multiple time variables [1], [6], to describe the multirate behavior. With this technique circuits are no longer described by ordinary differential systems in the 1-D time t but, instead, by partial differential systems. For example, if there are two separated rates of variation, then (1) is converted into a system of the form [1], [6]

$$\boldsymbol{p}\left[\hat{\boldsymbol{y}}(t_1,t_2)\right]+\frac{\partial \boldsymbol{q}\left[\hat{\boldsymbol{y}}(t_1,t_2)\right]}{\partial t_1}+\frac{\partial \boldsymbol{q}\left[\hat{\boldsymbol{y}}(t_1,t_2)\right]}{\partial t_2}=\hat{\boldsymbol{x}}(t_1,t_2). \quad (4)$$

In order to compute the multivariate solutions, some initial/boundary conditions must be added to (4). These are determined by the existence of periodicity in the signal components. Two typical cases of significant practical interest are: (i) the envelope modulated regimes and (ii) the quasiperiodic regimes. In the former we have an initial-boundary value problem while in the later we have a boundary value problem.

C. Envelope Transient over Shooting

The envelope transient over shooting [1], [4], is an efficient time-domain method that can be used to obtain the numerical solution of circuits described by initial-boundary value problems of the form of (4). This method is a particular implementation of a general technique that is often referred to as *envelope following* [14], and consists in replacing the derivatives of the slow aperiodic time scale (say t_1) by finite-differences approximations, to then obtain a set of successive boundary value problems with periodic boundary conditions,

$$\boldsymbol{p}\left[\hat{\boldsymbol{y}}_i(t_2)\right]+\frac{\boldsymbol{q}\left[\hat{\boldsymbol{y}}_i(t_2)\right]-\boldsymbol{q}\left[\hat{\boldsymbol{y}}_{i-1}(t_2)\right]}{h_{1,i}}+\frac{d\boldsymbol{q}\left[\hat{\boldsymbol{y}}_i(t_2)\right]}{dt_2}$$
$$=\hat{\boldsymbol{x}}(t_{1,i},t_2), \qquad \hat{\boldsymbol{y}}_i(0)=\hat{\boldsymbol{y}}_i(T_2), \quad (5)$$

where T_2 is the period in the periodic fast time scale t_2. Once $\hat{\boldsymbol{y}}_{i-1}(t_2)$ is known, the solution on the next slow time instant, $\hat{\boldsymbol{y}}_i(t_2)$, is obtained by solving (5). Each of the periodic boundary value problems of (5) is solved using the shooting method.

D. Envelope Transient over Shooting with MRK

Recently, a very powerful computer-aided design tool especially conceived for the efficient time-domain simulation of highly heterogeneous nonlinear RF circuits has been proposed [7]. This algorithm is based on an ingenious modification of the above described envelope transient over shooting technique. It splits the circuits into two distinct parts and, instead if using standard integrators, it uses modern multirate Runge-Kutta (MRK) schemes to perform time-step integration with different step sizes in each of the consecutive shooting iterations needed to solve (5). Thus, it can be seen as a multirate scheme (different time-step integration sizes to state variables that present significantly disparate rates of change) coupled with a multirate excitation regime (multiple time-scale representations). This way, it is able to benefit from the circuits' heterogeneities and also from the stimuli time-rate disparities, to dramatically reduce the required simulation time.

E. Hierarchical Shooting

Hierarchical shooting (HS) [1] is the multivariate version of classical one-dimensional shooting, and it is an efficient time-domain technique that can be utilized to evaluate the solution of boundary value problems of the form of (4) (e.g., circuits under quasiperiodic regimes). In the 2-D case, hierarchical shooting consists in performing shooting in one dimension, say t_2, nested in the other dimension, t_1. This means that, by considering the semi-discretization of the domain in t_1 using a finite-differences scheme approximation, it goes through successive shooting iterations in t_2 for each one of the consecutive t_1 time instants (inner loop). Then, it compares and updates the initial solution on the first t_1 time instant, according to the solution obtained on the last one, to afterwards repeat all the t_2 shooting process, until a bi-periodic solution is achieved (outer loop).

F. Multiple-Line Double Multirate Shooting

An innovative multiple-line shooting technique that solves 2-D bi-periodic boundary value problems even in a more efficient way than the hierarchical shooting has been recently developed [8]. This multiple-line shooting technique is based on the mathematical method of lines and, although operating in 2-D domains, it requires only one shooting loop, and so a reduced computational effort. In addition, because it introduces a multipartitioning strategy, which automatically classifies the circuit's state variables according to their time rates of change, it allows the simulator to take advantage of the latency of some state variables of the circuit in t_1, in t_2, or in both dimensions.

IV. Comparison Between Methods

In order to provide a realistic idea of the potential of the methods described in this paper, we decided to include a comparison between the univariate and bivariate techniques, in terms of computational speed. We considered the RF polar transmitter PA described in [7] as our illustrative application example, in which numerical computation times for two different simulation time intervals are presented in Table I. These results were obtained using: (i) a SPICE-like engine, (ii) a time-step integrator based on MRK schemes, (iii) an envelope transient over shooting (ETS) algorithm and (iv) a ETS algorithm with MRK. Since a careful comparison between HS and multiple-line shooting was already done in [8], we decided not to include such methods in this section.

TABLE I
COMPUTATION TIMES - SIMULATION OF A RF POLAR TRANSMITTER PA

Simulation time interval	Univariate time-step integration		Bivariate envelope following tools	
	RK based (SPICE)	MRK based	ETS	ETS with MRK
[0, 250 ns]	1h 1m 15s	12m 29s	7m 5s	38 s
[0, 1.0 μs]	> 5h	56m 45s	33m 22s	3m 14s

By comparing the results presented in Table I we can attest the efficiency gains provided by the use of different step sizes (MRK schemes) within the classical univariate time-step integration, but mostly within the bivariate formulation. We can also attest the advantage of using multiple time variables to describe the multirate behavior, i.e., the efficiency gains that can be achieved by converting ordinary problems running in multiple time scales into multi-dimensional problems.

V. REMAINING CHALLENGES

Although significant advancement has been made in time-domain simulation in the recent years, there are still several outstanding challenges in this research field. In the following, we will summarize some of them.

An important area where further work is needed is the analysis of circuits containing multi aperiodic regimes of widely separated time scales. It is so because the powerful multivariate strategy based on the partial differential formulation becomes useless when multirate signals do not evidence any periodicity in their components [1]. As a consequence, all the recently proposed advanced techniques described above become inefficient. For example, if we have a circuit whose state variables evolve according to distinct aperiodic regimes of widely separated rates of change, and the fastest regime is present in all the circuit's state variables, then we have no choice than to use a classic uni-rate SPICE-like engine to compute its solution in the one-dimensional time. Indeed, in such case, it is not even possible to use a MRK scheme for performing the time-step integration of the DAE system describing the circuit's operation. Similar difficulties arise when one, or more, periodic regimes are also added to the circuit. For instance, if an RF carrier of 2 GHz frequency, affected by low frequency phase noise of 100 kHz bandwidth, is modulated by a 2 MHz baseband information signal in some RF circuit, it is not possible to adopt a 3-D formulation for the simulation of that circuit because the two aperiodic entities (the 100 kHz phase noise and the 2 MHz baseband signal) have to be mixed in the same time dimension.

Another situation where further research may be of interest is the simulation of circuits whose state variables are all fluctuating in a fast varying time scale, but evidence different time evolution rates in a slow envelope time scale. An efficient computer-aided design tool especially conceived for that purpose is still missing. An attempt to solve this problem is described in [15], but the promising efficiency of the proposed method was constrained by a stiffness restrictive scenario.

Finally, the difficulty imposed by the number of time scales involved in the same problem leads to a limitation shared by both 1-D and multivariate methods. For instance, if we have a multitone regime with several tones, the computational effort required to simulate the circuit will tend to rise exponentially with the number of tones (time scales) involved. While in the frequency domain this difficulty can sometimes be overcome with the use of artificial frequency mapping [16], when we operate strictly in the time domain there is no tool capable of dealing with this kind of problems in an efficient way.

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